

Original Research Paper

Explosion of the Science Modification of Newtonian Dynamics Via Mach's Inertia Principle and Generalization in Gravitational Quantum Bound Systems and Finite Range of the Gravity-Carriers, Consistent Merely On the Bosons and the Fermions

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Abstract: Newtonian gravity is modified here via Mach's inertia principle (inertia fully governed by universe) and it is generalized to gravitationally quantum bound systems, resulting scale invariant fully relational dynamics (mere ordering upon actual objects), answering to rotation curves and velocity dispersions of the galaxies and clusters (large scale quantum bound systems), successful in all dimensions and scales from particle to the universe. Against the Milgrom's theory, no fundamental acceleration to separate the physical systems to the low and large accelerations, on the contrary the Newtonian regime of HSB galaxies sourced by natural inertia constancy there. All phenomenological paradigms are argued here via Machian modified gravity generalized to quantum gravity, especially Milgrom empirical paradigms and even we have resolved the mystery of missing dark matter in newly discovered Ultra Diffuse Galaxies (UDGs) for potential hollow in host galaxy generated by sub quantum bound system of the globular clusters. Also we see that the strong nuclear force (Yukawa force) is in reality, the enhanced gravity for limitation of the gravitational potential because finite-range of the Compton wavelength of hadronic gravity-carriers in the nucleuses, reasoning to resolve ultimately, one of the biggest questions in the physics, that is, so called the fine structure constant and answering to mysterious saturation features of the nuclear forces and we have resolved also the mystery of the proton stability, reasonable as a quantum micro black hole and the exact calculation of the universe matter. Tully-Fisher and Faber-Jackson relations and Fish's and Freeman's laws of the constant central gravitational potential and universalization of Baryonic Fish's law are next paradigms argued here. We don't play with mathematical functions to set them with empirical results and we don't simulate the models but the Newton's empirical gravity is returned to its fundamental face logically.

Keywords: MOND, quantum bound systems, Quantum Gravity, Mach's Inertia Principle, Dark Matter, Tully- Fisher Relation, Fish and Freeman's Laws, Yukawa Strong Force

1 Introduction

Albert Einstein based the theory of relativity regarding to Mach's mechanics (1960) as noted by Mach that "No one is competent to predicate things about facts. Absolute space and absolute motion; they are pure things of thought, pure mental constructs, that cannot be

produced in experience. All our principles of mechanics are, as we have shown in detail, experimental knowledge concerning the relative positions and motions of bodies."

For knowledge of Machian beautiful mechanics, one may refer to a list of Machian laws (Bondi and Samuel, 1997) referred to the book "Mach's Principle from Newton's Bucket to Quantum Gravity" (Barbour and

Pfister, 1982) so that: Mach0: The universe, as represented by the average motion of distant galaxies, does not appear to rotate relative to local inertial frames. Mach1: Newton’s gravitational constant G is a dynamical field. Mach2: An isolated body in otherwise empty space has no inertia. Mach3: Local inertial frames are affected by the cosmic motion and distribution of matter. Mach4: The universe is spatially closed. Mach5: The total energy, angular and linear momentum of the universe are zero. Mach6: Inertial mass is affected by the global distribution of matter. Mach7: If you take away all matter, there is no more space. Mach8: $\Omega = 4/3\pi G\rho t^2$ is definite number of order unity (ρ is universe mean density and t is Hubble time). Mach9: The theory contains no absolute elements. Mach10: Overall rigid rotations and translations of a system are unobservable.

In Newtonian mechanics, the inertia is an intrinsic property of the matter but in the Mach’s mechanics, the inertia is fully extrinsic as noted by Sachs (2003) that:

” The latter is the assertion that only the distant stars of the universe determine the mass of any local matter. In contrast to this, in his Science of Mechanics (1883), Mach said that all of the matter of the universe, not only the distant stars, determines the inertial mass of any localized matter. Nevertheless, it was Mach’s contention that in principle all of the matter of the closed system-the nearby as well as far away constituents-determines the inertial mass of any local matter.”

Mach recognized that the Newtonian mechanics requires modification, to be fully real and relational (mere ordering upon actual objects). Einstein (1918) via a preliminary scalar theory of gravitation found that the presence of the spherical shell of mass M and radius R increases inertial mass m of the point at its center as:

$$\frac{GM}{c^2 R} = 1 \quad (1)$$

Since we consider M and R as the mass and radius of the universe, Equation (1) is the so called Machian relation (Einstein-Dirac-Whitrow-Randall-Sciama-Brans-Dicke relation) which is the mathematical context of Mach’s inertia principle also extracted in different ways by some scientists.

Then Einstein introduced Mach’s Principle (1912) “... the presence of the inertial shell K increases the inertial mass of the material point P within it. This makes it plausible that the entire inertia of a mass point is the effect of the presence of all other masses, resulting from a kind of interaction with them. This is exactly the

standpoint for which E. Mach has argued persuasively in his penetrating investigations of this matter”.

This means that g_{mn} are completely determinable by the mass of bodies, more generally by T_{mn} .

But Einstein general relativity, was not success to obtain a fully connection to Mach’s inertia principle and Einstein field equation (Einstein, 1959) results partial dependency to Mach's inertia principle as Einstein in a letter to de Sitter 1917 states:

“in my opinion it would be dissatisfying, if there were a conceivable world without matter. There should rather be determined by the matter and not be able to exist without it. This is the heart of what I understand by the demand for the relativity of inertia. One could just as well speak of the ‘material conditionedness of the geometry’. As long as this demand was not fulfilled, for me the goal of general relativity was not yet completely achieved.”

Some scientists tried to reformulate the Einstein total field to obtain a kind of equation, compatible with Machian relation. Brans and Dicke (1961) reformulated Einstein general relativity via scalar tensor theory of gravitation, apparently compatible with Mach’s inertia Principle. Hoyle and Narlikar (1964; 1966) developed a theory of gravitation in context of the Mach’s inertia principle and they used the waves to communicate gravitational influence between particles. Another form, for Machian relation, called in the literature as Whitrow-Randall relation (Whitrow and Randall, 1951) and Sciama (1953) used electrodynamic type equations for gravity to extract the Machian relation as the Sciama’s law of the inertial induction.

But here we want to modify directly the Newtonian dynamics by Mach’s inertia principle to show a real alternative for discrepancies to show the fundamental face of the Newton's empirical law. We show here that the Milgrom’s MOND (Milgrom, 1983a; 1983b; 1983c) is an approximate empirical simulation, showing that Milgrom's constant acceleration a_0 is not fundamental. Mordehai Milgrom theorized a phenomenological equation instead Newtonian force as his model’s principal paradigm, claiming a new fundamental constant acceleration a_0 to split the physical systems to the low and large accelerations via a discrete formula as follows:

$$\begin{cases} a \leq a_0 \Rightarrow g^N = a^2 / a_0 \\ a > a_0 \Rightarrow g^N = a \end{cases} \quad (2)$$

So that g^N is Newtonian gravitational intensity and a is acceleration. This equation is mainly about to fit with

rotation curve and velocity dispersion of the galaxies. But Milgrom's formula for $a < a_0$ is inverse engineering of the Tully Fisher relation as in the below proposition we see that:

$$g = \frac{a^2}{a_0} \Leftrightarrow v_r^4 = a_0 GM_{<r} \quad (3)$$

We see that the Milgrom's gravity formula and Tully-Fisher relation are transferable directly to each other. However, Milgrom's formula was rearranged by Poisson type gravity "AQUAL" (Bekenstein and Milgrom, 1984) as:

$$\nabla \cdot \left[\left(g / a_0 \right) \vec{g} \right] = -4\pi G \rho \quad (4)$$

But this is replacement of Newtonian g^N in Poisson equation of gravity, with g^N in Milgrom's formula as a proof less sentence. Of course as Sanders and McGaugh (2002), one problem with AQUAL is failure to predict the amount of gravitational lensing actually observed in rich clusters of galaxies. On the other hand, Tully and Fisher (1977) and Faber and Jackson (1976) relations are not ever accurate in the galaxies, on the contrary the fake galaxies don't follow completely these relations. Then actually the Milgrom model of modified gravity is deviated from the reality in fake galaxies. Milgrom *et al.*, have tried to compensate failures with change of the uncertain parameters similar to mass to light ratio and distance uncertainties but there are serious difficulties, especially in the Bullet clusters and galaxies clusters and globular clusters (Jordi *et al.*, 2009; Baumgardt, 2006; Sollima and Nipoti, 2010; Aguirre *et al.*, 2001; Kent, 1987; Gentile *et al.*, 2011).

2 Modification of Newtonian Dynamics by Mach's Inertia Principle

In Mach's mechanics, the gravitational G is not constant but variable in agreement with Machian relation so that in an N active mass points m_k observable universe ($k=\{1,2,3,...N\}$) around a passive mass point m_p (everything has geometry but we assume here the mass points for simplification), whether universe occupying relative configurations (Barbour, 2001) or ether universe, for gravitational G in passive mass point m_p we have:

$$\frac{1}{G} = \frac{1}{c^2} \sum_{k=1}^N \frac{m_k}{r_{kp}} \quad (5)$$

The r_{kp} is distance of the masses from the passive mass point and integration is on the whole existing matter limited naturally to the relative Hubble sphere.

This is Machian relation (Einstein-Dirac-Whitrow-Randall-Sciama-Brans-Dicke relation) and in a continuum mode, Machian relation is written in the integral format on the universe content as:

$$G \frac{\int_U \frac{\rho_r}{r} dV}{c^2} = 1 \quad (6)$$

So that r is distance of the active mass points from the passive mass point and c universally is the light speed and G is gravitational coefficient at position of the passive mass point and ρ is medium density there. Then G satisfies a Machian law that

- "Mach1: Newton's gravitational constant G is a dynamical field."

We need to notify that the Machian mechanics is working on the observable universe and then the particles can't follow the imaginary conceptual parameters. Particles have their physical language (body language). Since humans are all sleeping, the universe is working yet. In the role of Mach's mechanics we can say that the universe is working on the whole spectrum, using a complete mechanical intelligence via material mind. In fact it was initially proposed by mystic and philosopher, George Ivanovich Gurdjieff that:

"In the universe everything is material and for this reason the Great Knowledge is more materialistic than materialism itself."

Of course we need to notice that George Gurdjieff statement is a Machian idea as noted by Mach that:

"But we must not forget that all things in the world are connected with one another and depend on one another and that we ourselves and all our thoughts are also a part of nature."

In reality the Mach's principle is a philosophy which the physics is following it.

Then since we speak out about the distance r from the passive mass point which we want to calculate the gravitational G in that point, r is not determined by pure frame against the Newton's absolute space because the Machian mechanics should be material.

On the other hand, there is no absolute simultaneity in the Machian mechanics and r is what we observe it, out of consideration of its creation time as a statement for Copenhagen interpretation. For example, a star in 14

billion light years, it is present for observer in the language of particles. Then for calculation of gravitational G in each point we need to use observed values directly, however absolutely non simultaneous. In fact, using standard units into the fundamental physics is a big error; however the units are suitable for human applications. Then it is predictable that the Mach's mechanics should be certainly non scale.

It is soon yet to understand accurately the Machian universe for that the Machian universe for its perfect physical mechanism is very complicate. For example the particles have physical intelligence so that Inverse Square is understood by physical reality that the wave flux is Inverse Square and the waves are transferring the data physically in Mach's mechanics. This mimics the transformation of the data physically by arrangement of the atoms in the DNA in biology.

In fact it is a question that is it possible a perfect Machian universe? Or ultimately a composite of physical and conceptual! Of course, conceptions are too real in higher dimensions as stated by Ernst Mach and George Gurdjieff. Maybe this question to be equivalent with Gödel's incompleteness theorems (1931) in mathematics and equivalent with a problem in biology called Darwinism.

By the way since we understand how the potential parameters should be considered in Mach's mechanics we continue the process logically here.

Replacing G in Newtonian gravity with variable G in Machian relation returns it from local mutual empirical type to its fundamental universal primary face that:

$$g^N = \frac{G_N \sum_{k=1}^N \frac{m_k}{r_{kp}}}{c^2} a \quad (7)$$

This relation is what the Brans (1961) "Mach's principle and a varying gravitational constant" did extract it by dimensional calculus, of course Carl Brans decided to continue on the modified general relativity instead modified Newtonian gravity and also this relation is the same Einstein-Sciama force, however Einstein-Sciama force is assumed as an inertial reaction force. Berry (1989) did result this equation by next way the Sciama named it, the law of inertial induction and, a next general way to extract the Machian relation is the way of zero total energy (Filippenko and Pasachoff, 2010; Krauss, 2009; Berman, 2007; 2008; 2009). Some other scientists also have tried in this way and there are many papers not possible to cite all here. All these ways reach to an answer called generally "Machian relation".

The potential associated with a mass distribution is the superposition of the potentials of point masses. If the mass distribution is a finite collection of point masses, and if the active point masses are located at the points x_1, \dots, x_N and have masses m_1, \dots, m_N , then the potential of the distribution at the passive mass point x_p is

$$\varphi_{x_p} = \sum_{k=1}^N \frac{Gm_k}{|x_k - x_p|} = r_{kp} \quad (8)$$

The gravitational potential is active and separable whereas the potential of distribution at the Machian relation is passive and not separable. Then we call it here the inertial potential. However the gravitational potential and inertial potential are usually equal but we will see that in quantum bound systems included to sub quantum bound systems, the gravitational potential and inertial potential are not equal. Then since lowercase phi (φ) is used for gravitational potential, then to realize between gravitational potential and inertial potential we use here from uppercase phi (ϕ) for inertial potential and also we use the symbol psi (ψ) for G-independent potential of the distribution as the integral on the relevant green function $1/|x_k - x_p|$ so that:

$$\psi = \sum_{k=1}^N \frac{m_k}{r_{kp}} \quad (9)$$

Then the Equation (7) simply is written as follows:

$$g^N = \frac{\phi}{c^2} a \quad (10)$$

And then we can define the inertia called here i in the Newtonian law of the inertia as:

$$g^N = i \times a \quad i = \frac{\phi}{c^2} \quad (11)$$

The inertia in a language is defined as

$$i = \frac{m_a}{m_p} \left| g^N m_g = m_a a \right. \quad (12)$$

So that m_g is defined as the gravitational mass and m_a as the inertial mass. But these are relative definitions whereas the mass is the mass only and what it has been defined in the term of inertial mass m_a it is not the mass but multiplication of i to the mass of the passive body whereas the gravity is not dependent to the mass of passive body as visible in the Equation (11) and the parameter i is just a coefficient for the law of inertia. But relatively it is not paradox to define imaginary mass called inertial mass as follows:

$$m_a = im_g \quad (13)$$

However such a definition contradicts the Einstein equivalence principle because that on the Einstein equivalence principle as the equality of gravitational and inertial masses, i should be ever equal to 1 whereas that $i=1$ theorem contradicts the universality of the Newtonian G_N because that according to the Machian relation (equation 6), $i = 1$ is verified just in variable G universe.

Then inertia here satisfies the Machian laws mentioned in the list that:

- "Mach2: An isolated body in otherwise empty space has no inertia."
- "Mach6: Inertial mass is affected by the global distribution of matter."

The inertia i and gravitational G are both the coefficients in the law of inertia and if we assume the inertia i as a variable, then G would be constant and inverse. This modified gravity is a scale-invariant relation, invariant by the meter and second and kilogram units because if k and k' and k'' to be assumed arbitrary coefficients and assuming Equation (11) in a general function f , then we have:

$$f(kx, k't, k''m) = 0 \quad (14)$$

And this is the magic of modified gravity by Mach's inertia principle showing that the fundamental relations in the physics are non-scale whereas the Newton's gravity is not invariant by units' transformation. The language of tensors is non-scale, but in solution of the Einstein field equation, scientists have used absolutist boundary conditions, transferring the Einstein relation to scalar format and this is the answer for question why the Einstein field equation ultimately not match with Machian mechanics.

In a rigorous criterion, the Laue (1921, P. 180) discussed the Einstein general relativity as noted that:

"according to the fundamental idea of the general theory of relativity, the inertia of a single body should vanish if it is at a sufficient distance from all other masses. for inertia can only be a relational concept, which can be applied only to two or more bodies. ... with the boundary conditions mentioned, however the inertia continues to exist. Such considerations have led Einstein to the hypothesis of a space which runs back on itself like the surface of a sphere."

In the Milgrom's MOND too, his formula is scale-invariant in accelerations $a < a_0$, where a_0 corresponds

to Milgrom's fundamental acceleration (Milgrom, 2009), however being non-scale in a domain and scalar in the next domain is manifestly showing the failure of Milgrom's MOND.

Dicke (1962) also did discuss on scale-invariant gravity in relativistic context as noted by R. Dicke that:

"... It is evident that the particular values of the units of mass, length and time employed are arbitrary and that the laws of physics must be invariant under a general coordinate-dependent transformation of units."

And also scale-invariant gravity has been modelled in shape dynamics by (Barbour and Bertotti, 1982; Barbour, 2003).

3 Generalization of Modified Gravity in Quantum Bound Systems (QBS)

3.1 The Boundary of Gravitationally Quantum Systems

One of the Mach's principles as listed by Bondi and Samuel (1997) is that:

- "Mach4: The universe is spatially closed."

As noted by Barbour (2010):

"However, the Machian view point is only possible if the universe is a closed dynamical system. I shall say something about the possibility of a truly infinite universe at the end of this paper. If we do suppose that the universe is a closed system, we can attempt to describe it by means of a relational configuration space obtained by some quoting with respect to a group of motions."

Of course, a truly infinite universe is impossible in Mach's universe, for that the infinity is not observable as Hilbert *et al.* (2013) famously argued that infinity cannot exist in physical reality and Dennis Sciama adopted a statement (Sciama, 1964) that:

"Inertial forces are exerted by matter, not by absolute space. In this form the principle contains two ideas:

(1) Inertial forces have a dynamical rather than a kinematical origin and so must be derived from a field theory [or possibly an action-at-a-distance theory in the sense of J.A. Wheeler and R.P. Feynman...]

(2) The whole of the inertial field must be due to sources, so that in solving the inertial field equations the boundary conditions must be chosen appropriately."

In relativistic context too, Machian universe is a quantum bound system as noted by Ghosh (2002) that:

“Thus we see here how the Mach principle is entirely intertwined with the theory of general relativity, regarding the logical dependence of the inertial mass of local matter on a closed system.”

Then in Machian frame work, the inertial field equations should be solved merely in a definite shape of the universe we name it here the quantum bound system, which is, the gravity is interconnected to mass point particles limited to a natural quantization of material system. Also it is observed that the galaxies and galaxies clusters are inertial stationary and atoms and stars are not expanded by space expansion. This is an evidence for reality that the galaxies and galaxies clusters are gravitationally quantum bound systems. In relativistic universe, every point is center of a relative Hubble sphere and then the gravity needs for a quantum bound system as discrete scale relativity until to be possible the galaxies and clusters. Without quantum bound system gravitation, it is impossible rotation curves. Noticing that cosmologists distinguish between the observable universe and the entire universe, the former being a spherical portion of the latter that can, in principle, be accessible by astronomical observations. Neither Newton gravity nor Einstein gravity has any possible solution for the fractal separation in the scale of the cosmic structure and hierarchical structure of the cosmic is a manifest evidence for quantum bound systems.

Then the question is that what defines the boundary of inertial field equations?

Mach’s mechanics is a fully relational dynamics and then the boundary of inertial field equations is defined inertial relationally where the internal inertia is larger than the external inertia as the dominance law of the gravitational quantum bound systems. This means mathematically that:

$$i_{int} > i_{ext} \quad (15)$$

This law of dominance quantizes the universe to discrete hierarchical systems. In fact the universe is very similar to the pomegranate. Of course we need to notice that the observable universe is not a gravitationally quantum bound system but the cosmos observationally limited to Hubble sphere. Integration is limited to the Hubble sphere for Machian relation because the universe expansion doesn’t allow interring even the light.

3.2 Generalization of the Machian Relation in the Gravitationally Quantum Bound Systems

By gravity limitation to the boundaries we need to integrate the inertia on the enclosed mass of the system as association of all mass point particles m_k under

dominance of the Quantum Bound System (QBS) so that from Equations (10) and (15) we have:

$$g^N = \frac{G_N \sum_k \frac{m_k}{r_{kp}}}{c^2} a |m_k \in QBS \quad (16)$$

The constant c of the quantum bound system is not light speed unless in the entire observable universe suppose it is system depended constant, variable from system to system. For asymptotic velocity in the infinity $r = \infty$ it is deduced by Equation (16) that:

$$C = v_\infty \quad (17)$$

But v_∞ is itself a result derived from c and then to determine c it needs extra information. In fact c is a degree of freedom for quantum bound system and anything can’t determine it inside of the system. The c is a degree of freedom to determine G in the quantum bound system and then determination of the G for quantum bound system is equivalent with determination of c . The G in the system in different points is not constant and then G in the rest of the system which is invariant by system is c equivalent.

Then $G(0)$ is a degree of freedom for Quantum Bound System (BQS) and it can’t be determined internally but it should be determined by host server and consequently extrapolating yields to the observable host universe as a reference for $G(0)$ in quest sub systems. Then universe G_u in the Machian relation is the reference for $G(0)$ in the quest quantum bound systems, that is:

$$G_u = G_{QBS}(0) \left| \frac{G_u}{c^2} \sum_u \frac{m}{r} = 1 \quad (18)$$

In the present cosmic time, the solar system is positioned at the Newtonian regime of the Milky Way galaxy and then we have:

$$G_U = G_N \quad (19)$$

Then presently, the Newtonian gravity is universal for rest of the gravitational quantum bound systems. But since the Earth leaves the Newtonian regime, the gravity at the Earth too, will deviate from universal G -value. In fact, the G in the quantum bound systems is dependent to the G in the rest of the system and G in the rest of the system is dependent to the G of the relative observable universe. Then the gravitational quantum bound systems are inertial stationary for their centers, verifying not expanding by space expansion as the ships in the ocean. By the way, Machian relation is compatible for Hubble sphere and substituting the universe mass and the radius

of Hubble sphere into the Machian relation verifies it. As argued in (Lutephy, 2019a), the Planck scale quantum system is a mini universe and Machian relation too is compatible there.

The dependency of the systems to the largest observable universe has been mentioned as a statement of Mach's principle by Barbour (2010) that:

"The application of Mach's principle is to be considered whenever direct observations or theoretical considerations suggest that the physical configuration space of a closed dynamical system is to be obtained by group quotienting of a larger configuration space that contains redundant information unobtainable by direct observation."

Each quantum bound system is gravitationally an independent system (quasi-universe) that the gravity is defined there by a variable-G which has a degree of freedom in the rest of the system (center of mass invariant by physics laws) compatible with Mach's principle as noted by Sachs (2003) that:

"The dependence of the inertial mass of localized matter, in particular, on the rest of the matter of the 'universe', is a statement of the Mach principle."

Then gravitational quasi universes (quantum bound systems) in Machian mechanics follow whole possible observable universe, satisfying one of the Machian paradigms listed in (Bondi and Samuel, 1997) that:

- "Mach3: Local inertial frames are affected by the cosmic motion and distribution of matter."

Then by Equations (17, 18, and 19) in quantum bound systems we have:

$$v_{\infty}^2 = G_N \sum_{k=1}^N \frac{m_k}{r_{ko}} \quad (20)$$

This is generalized Machian relation in quantum bound systems as a strong verification for MMOND. Multiplication of the Equation (20) to the passive mass point m_p yields to:

$$m_p v_{\infty}^2 = G_N \sum_{k=1}^N \frac{m_k m_p}{r_{ko}} \quad (21)$$

Interestingly, for flattening of rotation curves, the Equation (21) verifies the Virial theorem that $E_p = 2E_K$! Noticing that Virial theorem does not depend on the

notion of temperature and holds even for systems that are not in thermal equilibrium. Of course, in the section 14, the Equation (21) is derived for velocity dispersion too. Then by Equations (15, 16, 20) it is deduced:

$$g^N = \frac{\phi_r}{\phi_0} a_r \quad (22)$$

This equation is also written as:

$$g^N = \frac{\sum_{k=1}^N \frac{m_k}{r_{kp}}}{\sum_{k=1}^N \frac{m_k}{r_{ko}}} a_r \quad (23)$$

Newtonian G is universal presently and then substituting Newtonian G from Equation (6) into the relation (23) for an isotropic system yields to:

$$\frac{M_{<r} c^2}{r^2} = \sum_u \frac{m}{r} \frac{\sum_{k=1}^N \frac{m_k}{r_{kp}}}{\sum_{k=1}^N \frac{m_k}{r_{ko}}} a_r \quad (24)$$

The c is light speed here and this relation is non-scale and fundamental version of Newtonian empirical gravity.

Rarely in some area it is possible that $\phi_r > \phi_0$ and then in some specific areas it is considerable fictitious dark matter with negative density, for example in some areas between two very near galaxies, as noted in one of the Milgrom paradigms (Milgrom, 1986a) that:

- "A DM interpretation of MOND should give negative density of "dark matter" in some locations (Milgrom 1986a)."

Also the shape of fictitious dark matter follows the inertial potential in the galaxies and despite the mass integration, the ϕ is not constant in both projected and de-projected density profiles. Then in the model of the fictitious dark matter it needs ever to use a disk component for fictitious dark matter and this is one of the next paradigms of the Milgrom's MOND that:

- "Disc galaxies are predicted to exhibit a disc mass discrepancy. In other words, when MOND is interpreted as DM we should deduce a disc component of DM as well as a spheroidal one (Milgrom, 1983b; 2001; Famaey and McGaugh, 2012)."

4 The Fishian Gravitational Systems

4.1 Fish's Law and Freeman's Law

For a sample of about two dozen elliptical, Fish (1964) discovered a relationship between the total potential energy and total mass of the galaxies. Subsequently, Carolo *et al.* (1997) showed that, within the general class of pressure supported systems, there appears to be characteristic surface brightness, which is on the order of that implies by Fish and therefore Fish law was recovered for the larger set of pressure-supported objects.

Freeman (1970) commented that Fish's law can also be interpreted more directly to state that central surface densities of the Fish's sample of ellipticals has a universal value. According to the Freeman's law in disk galaxies (Freeman, 1970), the central surface brightness of a spiral galaxy disk, appears to be constant from galaxy to galaxy, but the scale length h varies. In other words, all spiral galaxies seemed to have about the same central surface brightness, but they vary in size.

On the base of the observable mass of the galaxies with different morphologies we observe that the Fish's law is not specified to the elliptical galaxies suppose the galaxies all are scattering around the Fish's law so that newly Freeman's law was confirmed for a sample of 30000 Sloan Digital Sky Survey galaxy images (Fathi, 2010) or we may refer to the paper (Allen and Shu, 1979) against the selection effect (Disney, 1976) so that Rolland Allen and Frank Shu demonstrated that this selection actually works only in one direction that there is indeed an upper limit to the surface brightness of the galaxies.

According to the Fish's law, for total mass M of the galaxies in relation with total potential energy we have:

$$W = 9.6 \times 10^{-11} M^{3/2} \quad (25)$$

W is associated total potential energy of the system calculated from visible mass.

On the definition of total potential energy, Fish's law is also written as follows:

$$\sum_{k=1}^N \sum_{j < k} \frac{Gm_k m_j}{r_{jk}} = 9.6 \times 10^{-11} M^{3/2} \quad (26)$$

In agreement with total zero energy as a way of Machian relation we have:

$$\sum_{k=1}^N \sum_{j < k} \frac{Gm_k m_j}{r_{kj}} = G \sum_{j=1}^N m_j \times \sum_{k=1}^N \frac{m_k}{r_{ko}} \quad (27)$$

The r_{k0} is distance of the masses from rest of the system. Substituting relation (27) in the Equation (26) yields to a relation as follows:

$$\sum_{k=1}^N \frac{m_k}{r_{k0}} = \sqrt{2M} \quad (28)$$

Then we obtain central potential ψ_o that:

$$\psi_o = \sqrt{2M} \left| \psi_o = \sum_{k=1}^N \frac{m_k}{r_{ko}} \right. \quad (29)$$

The Equation (29) is an equivalent version of the Fish's law and generally the galaxies are scattering around the Equation (29). We call this new relation (Equation 29), the "Fishian relation".

But we can consider a scattering coefficient λ' so that:

$$\psi_o = \lambda' \sqrt{2M} \quad (30)$$

And as a definition here, a gravitational system is Fishian if:

$$\lambda' = 1 \quad (31)$$

And if not Fishian, we call it the fake here.

4.2 Tully-Fisher Relation Derived by Modified Gravity in Gravitationally Quantum Bound Systems

Tully-Fisher relation is an empirical relationship between the mass or intrinsic luminosity of a spiral galaxy and its asymptotic rotation velocity or emission line width. It was first published by astronomers, Tully and Fisher (1977).

By Equation (16) and mixing it with Equation (19) it is extracted that:

$$v_\infty^2 = \varphi_0 \quad (32)$$

This relation is generalization of the Tully-Fisher relation in Machian mechanics as a correlation between central inertial potential and square of asymptotic rotation curve of the galaxies. We should notice that gravitational potential and inertial potential are equal usually.

By Equation (30) substituting into this Equation (32) it is deduced below proposition:

$$\psi_o = \lambda' \sqrt{2M} \xrightarrow{MMOND} v_\infty^4 = \lambda'^2 2G_N^2 M \quad (33)$$

Since we have an assumed Fishian galaxy ($\lambda' = 1$), then by Equation (33) it is deduced Tully-Fisher relation resulting too that the Tully-Fisher normalization factor k ($v_\infty^4 = kM$) is equal to $G_N^2 M$ so that:

$$v_\infty^4 = 2G_N^2 M \quad (34)$$

The proposition (33) is showing also the correlation between Fish's law and Tully-Fisher relation. We can see that Tully-Fisher relation is compatible conditionally if the Fish's law is compatible. The proposition (33) is a

strong verification for Machian MOND (MMOND) and verification for generalized Tully-Fisher relation.

4.3 Sersician Systems are Fishian

4.3.1 Independency of Central Surface Density from Effective Radii in Disks (Generalization of Freeman's Law)

In reality the galaxies, ideally want to obey the Sersic profile that:

$$\Sigma = \Sigma_0 \exp\left\{-\left(R/h\right)^{1/n}\right\} \quad (35)$$

So that R is radii in the disk and h is scale length and n is Sersic number and Σ_0 is central surface density. Calculation of the total mass by this profile yields to:

$$M = 2\pi \Sigma_0 \int_0^\infty \exp\left\{-\left(R/h\right)^{1/n}\right\} R dR \quad (36)$$

But to calculate central gravitational potential or inertial potential in Sersic profile, despite the mass calculation it requires separating systems to the two different types. A type is spherical and a type is disk and initially we consider that the system to be flat and agreement with Sersic profile.

In the flat type systems, to calculate central inertial potential we need to use Sersic profile as:

$$\varphi_0 = G \sum_{k=1}^N \frac{m_k}{r_{ko}} = 2\pi G \Sigma_0 \int_0^\infty \exp\left\{-\left(R/h\right)^{1/n}\right\} dR \quad (37)$$

Then to agree with Fishian relation (Equation 29) we have:

$$\varphi_0 = G\sqrt{2M} \quad (38)$$

$$\sqrt{\pi \Sigma_0} \int_0^\infty \exp\left\{-\left(R/h\right)^{1/n}\right\} dR = \sqrt{\int_0^\infty \exp\left\{-\left(R/h\right)^{1/n}\right\} R dR} \quad (39)$$

Perfect solution of this Equation (39) results below relation which it is independent of the scale length h :

$$\Sigma_0 = \frac{n}{\pi} \frac{\Gamma(2n)}{\pi(\Gamma(n+1))^2} \quad (40)$$

Gamma function is the so called gamma function in the mathematics and in the integer number N , the Equation (40) results:

$$\Sigma_0 = \frac{N}{\pi} \frac{((2N-1)!)}{(N!)^2} \quad (41)$$

This is generalization of the Freeman's law in Sersician flat type galaxies. Substituting Sersic indices $n = 1$ in this relation results:

$$\Sigma_0 = \frac{1}{\pi} \quad (42)$$

In magnitude arcsecond⁻² unit this is $\Sigma_0 = -21.65$.

This value of central surface density remembering the Freeman's law generalized here as a general continuum law of constant central surface density in disk type galaxies that ideally want to obey Sersic profile. This argument shows that the Fishian flat systems are all Sersician. The galaxies are not ever Sersician and actual galaxies are scattering around the ideal constancy of the extrapolated central potential because of deviations from ideal Sersic profile.

From Equation (41) for ideal disk galaxies with Sersic indices $n = 2$ it is given in magnitude arcsecond⁻² the size equal to $\Sigma_0 = -19.74$ and this is a prediction here for ideal disk galaxies with Sersic number $n = 2$. Then in ideal condition in the disk type galaxies, the central surface density is ideally independent of the effective radii but correlated to the Sersic number.

4.3.2 Independency of the Central Surface Density by Effective Radii in Spherical Galaxies (Generalization of Fish's Law)

The spherical type galaxies ideally have the density profile as Sersic profile similar to the flat type.

But difference is that in the spherical type galaxies, the Sersic profile is not real on the contrary it is a projected surface density from a real space density ρ_r .

The mass integration is not dependent to the type of the galaxy and either projected or deprojected, both are possible to be used in the mass calculation so that it is correct below relation that:

$$\int_0^\infty \rho_r dV = \int_0^\infty \Sigma_R dS \quad (43)$$

And then in spherical galaxy too, we can use the Sersic profile directly to calculate the mass as:

$$M = 2\pi \Sigma_0 \int_0^\infty \exp\left\{-\left(R/h\right)^{1/n}\right\} R dR \quad (44)$$

But to calculate central inertial potential in spherical type galaxies it is not correct using directly the Sersic profile into the integration and certainly the deprojected surface density should be transferred to the real volume density.

We can split the sphere to the n infinity number of slices by differential angle $d\theta$ so that:

$$2\pi = nd\theta|_{n \rightarrow \infty} \quad (45)$$

Each slice has an inertial potential equal with other slice for symmetry and because that the inertial potential in both volume integration and surface integration is the mass per the distance, then there is below correlation between volume integrated ϕ_ρ and projected surface integrated ϕ_Σ so that in a differential slice we have:

$$2d\phi_\rho = \cos\theta d\phi_\Sigma \quad (46)$$

And factor 2 in the Equation (46) it is reasoned here by the fact that deprojected potential, in each radius, is originated by θ and $-\theta$ in the sphere. Then if there is a projected profile so that central potential is not infinity, it is resulted that:

$$2\phi_{0,\rho} = \phi_{0,\Sigma} \int_0^{\pi/2} \cos\theta \frac{d\theta}{\pi/2} \quad (47)$$

$$\phi_{0,\Sigma} = \pi\phi_{0,\rho} \quad (48)$$

And this means that the central potential, integrated on the deprojected surface density is π times larger than that of real volume integrated shape of central potential.

The Sersic profile has a finite central potential and then the Equation (48) is agreement with that. Substituting the relation (44, 48) into the Fishish relation (Equation 29) results:

$$\int_0^\infty \frac{\Sigma_R}{R} dS = 2\pi \sqrt{\pi \sum_0 \int_0^\infty \exp\{-(R/h)^{1/n} R dR\}} \quad (49)$$

$$\int_0^\infty \exp\{-R/h\}^{1/n} dR = \sqrt{\pi \sum_0 \int_0^\infty \exp\{-(R/h)^{1/n} R dR\}} \quad (50)$$

$$\sum_0 = \pi \frac{n\Gamma(2n)}{(\Gamma(n+1))^2} \quad (51)$$

This is generalization of Fish's law in the spherical galaxies and for Vaucauleurs profile that $n = 4$ it is deduced in mag arcsecond⁻² the size equal to $\Sigma_0 = -14.59$ and this is remembering the Fish's law in ellipticals. Amazingly we see that the central surface density of the ideal spherical galaxies is π^2 times larger than the central surface density of the disk galaxies in each assumed Sersic number as a new law in astrology. Now we can predict the extrapolated central surface density of the spherical galaxies in Sersic indices $n = 2$ and embedding this number into the Equation (51) results $\Sigma_0 = 3\pi$ or in mag arcsecond⁻² that $\Sigma_0 = -17.25$ and to test this

prediction it needs to set sample of the galaxies with Sersic indices $n = 2$.

4.3.3 Argument of the Kormendy Relation

The perfect solution of Sersic profile deprojection is not a simple finite result and various expressions have been proposed as the approximation to the projected Sersic profile below:

$$\sum_R = \sum_e \exp\left\{-b_n \left[\left(R/R_e\right)^{\frac{1}{n}} - 1\right]\right\} \quad (52)$$

Using exact solution of the deprojected Sersic profile is not ever agreement actually. For example, in the Ciotti (1991), the deprojected profile for infinitesimal small radius r of the galaxy with Sersic number $n > 1$ is visible as:

$$\rho_r = \frac{\Sigma_0}{R_e} \frac{B[1/2, (n-1)/2n]}{2\pi n b_n^{-n}} \left(r/R_e\right)^{1-n/n} \exp\left\{-b_n \left(r/R_e\right)^{1/n}\right\} \quad (53)$$

So that B is a mathematical function and the density at the center is infinity and then disagreement actually and the numerical expression for deprojection of the Sersic profile results such infinity of density at the center, for example, referring to the Young (1976) verifies this. Then the independency of the central surface density from effective radii that it is a law generated from pure deprojection of the Sersic profile is not ever agreement actually. In the particular case of the de Vaucouleurs profile $n = 4$, Young (1976) performed a numerical deprojection and Mellier and Mathez (1987) presented a good approximation of this profile. Ciotti (1991) generalized these approaches and presented photometric and dynamical parameters for profiles of the type $r^{1/n}$, for n from 2 to 10. Here we refer to a newly wide used, Prugniel and Simien (1997) as a very good approximation of Sersic profile deprojection that it is in reality generalization of the Mellier and Mathez deprojection profile as:

$$\rho_r = \rho_0 \left(\frac{r}{R_e}\right)^{-p} \exp\left\{-b_n \left(\frac{r}{R_e}\right)^{1/n}\right\} \quad (54)$$

As noted in the Lima Neto *et al.* (1999) and Marquez *et al.* (2001), the total mass profile of the Prugniel-Simien model is given by below equation that:

$$M_r = 4\pi\rho_e \exp\{b_n\} R_e^3 n b_n^{n(p-3)} \Gamma(n(3-p)) \quad (55)$$

Equating total mass from Equation (55) to the surface-integrated total mass from its projected Sersic profile, the projected density Σ_0 is given by:

$$\Sigma_0 = 2\rho_0 R_e b_n^{n(p-1)} \frac{\Gamma(n(3-p))}{\Gamma(2n)} \quad (56)$$

And substituting this into the Equation (54) deduces:

$$\rho_r = \frac{\Sigma_0}{2R_e} \frac{1}{b_n^{n(p-1)}} \frac{\Gamma(2n)}{\Gamma(n(3-p))} \left(\frac{r}{R_e}\right)^{-p} \exp\left\{-b_n \left[\left(\frac{r}{R_e}\right)^{\frac{1}{n}}\right]\right\} \quad (57)$$

The term b_n is a function of n chosen so that R_e encloses half of the projected total galaxy light. It can be obtained by solving the expression (Ciotti, 1991) that:

$$\Gamma(2n) = 2 \times \gamma(2n, b_n) \quad (58)$$

The quantity Γ is the gamma function and γ is the incomplete gamma function given by:

$$\gamma(2n, b_n) = \int_0^x e^{-t} t^{a-1} dt, a > 0 \quad (59)$$

The quantity p is a function of n chosen to maximize the agreement between the Prugniel-Simien model and a deprojected Sersic model having the same parameters n and R_e . From (Lima Neto *et al.*, 1999; see also their Figure 13), a good match is obtained when p is equal to:

$$p = 1 - \frac{0.6097}{n} + \frac{0.05463}{n^2}, 0.6 < n < 10 \quad (60)$$

This profile is adequate to calculate central potential energy in galaxies for very wide rang agreement as:

$$10^{-2} < r/r_e < 10^3 \quad (61)$$

In the prugniel-Simien deprojected profile we see that the density at the center is again infinity so that:

$$r \rightarrow 0 \Rightarrow \rho \rightarrow \infty \quad (62)$$

This singularity is related to the below sentence that:

$$\rho_r \propto (r/R_e)^{-p} \quad (63)$$

This sentence causes to appear such a density which is infinity at the center and then this sentence is not agreement in the center actually. It requires a modification in the pure deprojected density profile.

Comparing the pure and actual density profiles is showing that the discrepancy between actual and pure values begins from the scale length of the galaxies and then to modify the Prugniel-Simien profile to avoid from a singularity at the center it requires to a sentence that below a transition radii r_1 (\sim scale length h), the density

be transferred gradually to a finite size in center and this can be modeled by a simple transformation as follows:

$$\left(r/R_e\right)^{-p} \rightarrow \left(\left(r_1 + r\right)/R_e\right)^{-p} \quad (64)$$

This transformation doesn't transfer the Prugniel-Simein profile for outer of the scale length h suppose generating a simple actual core model of the Prugniel-Simien to avoid from infinity of the profile at the center. Then we can reform pure profile of the Prugniel-Simein in Equation (57) to the actual profile below:

$$\rho_r = \frac{\Sigma_0}{2R_e} \frac{1}{b_n^{n(p-1)}} \frac{\Gamma(2n)}{\Gamma(n(3-p))} \left(\frac{r_1 + r}{R_e}\right)^{-p} \exp\left\{-b_n \left(\frac{r}{R_e}\right)^{1/n}\right\} \quad (65)$$

On the other hand, when we compare large values of the Sersic indices with small values, for example vaucouleurs profile comparing with exponential form $n = 1$ we see that in the vaucouleurs profile, the surface density in the center of the galaxy initially want to decrease highly but after the scale length h , the profile would to decrease slow whereas that in the exponential form $n = 1$ below the scale length h , the profile would to decrease slow versus the vaucouleurs profile but after passing from scale length, suddenly the profile would to decrease highly. Then for large values of Sersic indices, the outer regions of the scale length h are the more effective in the integral of central potential for proportionally slow mass distribution after scale length h so that for large values of Sersic indices it is agreement that:

$$\int_0^{r_1} \frac{\rho_r}{r} dV < \int_{r_1}^{\infty} \frac{\rho_r}{r} dV \quad (66)$$

But for a small size of the Sersic indices for example $n = 1$, the density outer of scale length h decreases highly proportionally whereas in the lower radii, below the scale length h , the density profile is almost uniform so that:

$$\int_0^h \frac{\rho_r}{r} dV \geq \int_h^{\infty} \frac{\rho_r}{r} dV \quad (67)$$

Then to calculate central potential in galaxies with low size Sersic indices it needs to use actual Prugniel-Simien profile as the Equation (66) that:

$$\rho_r = \frac{\Sigma_0}{2R_e} \frac{1}{b_n^{n(p-1)}} \frac{\Gamma(2n)}{\Gamma(n(3-p))} \left(\frac{r_1 + r}{R_e}\right)^{-p} \exp\left\{-b_n \left(\frac{r}{R_e}\right)^{1/n}\right\} \quad (68)$$

For simplification by definition of a coefficient k_n this relation is written as follows:

$$\rho_r = k_n \left(\frac{r_i + r}{R_e} \right)^{-p} \exp \left\{ -b_n \left(\frac{r}{R_e} \right)^{1/n} \right\} \quad (69)$$

Substituting this relation into the central G-independent gravitational or inertial potential results:

$$\int_0^\infty \frac{\rho_r}{r} dV = 2\pi \frac{\Sigma_0}{R_e} k_n \int_0^\infty \left(\frac{r}{R_e} \right)^{-p} \exp \left\{ -b_n \left[\left(\frac{r}{R_e} \right)^{1/n} \right] \right\} r dr \quad (70)$$

$$\int_0^\infty \frac{\rho_r}{r} dV = 2\pi \frac{\Sigma_0}{R_e} k_n R_e^p \int_0^\infty r \exp \left\{ -b_n \left[\left(\frac{r}{R_e} \right)^{1/n} \right] \right\} dr \quad (71)$$

$$\psi_0 = 2\pi k_n \Sigma_0 R_e R_e^p \quad (72)$$

Substituting this relation into the Fishian relation (Equation 29) results:

$$\psi_0 = \sqrt{2 \left(2\pi \Sigma_0 \int_0^\infty \exp \left\{ -b_n (R/h)^{1/n} \right\} R dR \right)} \quad (73)$$

$$\Sigma_0 \propto R_e^{-2p} \quad (74)$$

It was resulted above that the Sersic indices requires to be small and near to one and then we need to use $n \sim 1$ into the parameter p in the Equation (74) and then we have:

$$\Sigma_0 \propto R_e^{-0.8} \quad (75)$$

And this is remembering Kormendy relation (1977).

For Sersic indices noticeably larger than the $n \approx 1$, the outer regions inertial potential begins to be larger than that of inner regions and then the Kormendy relation is agreement the more for $n = 1$ type elliptical galaxies. Of course it needs here to assert that the Fishian relation is not ever independent of the external field effect suppose if the inner regions are star rich, then the number density of the globular clusters increases and this will affect the calculation of central inertial potential and too in the gas rich center galaxies, the density profile of the inner regions is not usual density profile but these galaxies would to shape an almost constant density core and then probably the more suitable, for appearance of the Kormendy relation, of course it needs to refer to exact analyzing of the systems according to Internal Field Dominance (IFD), what we will show it in the next sections. Then it is not ever to agree Kormendy relation in small Sersic indices but it needs to realize exactly the density profile of the galaxies actually and even for internal filed dominance, it is possible to appear Kormendy relation in very compact galaxies with large Sersic indices too.

What it is certain here it is that the exponential type galaxies $n = 1$ are agreement the more with Kormendy relation if those mass distribution be in the manner of Equation (67). In this condition we should await Kormency relation instead constancy of central surface density and then the core of the galaxies is very effective in the appearance of the Kormendy relation.

We see that the Fishian systems are all the systems which follow Sersic profile and then Fishian relation is in fact a law of Sersician systems. This means that in the systems following Sersic profile, we have $\lambda' = 1$. Then Tully-Fisher and Faber-Jackson relations are Sersician relations. It is clear that for non Sersician systems, the systems will deviate from Tully-Fisher relation, but amazingly the generalized Tully-Fisher relation (Equation 32) is yet invariant.

4.3.4 Dark Halo Constant Central Surface Density

To explain the failure of the rotation curve, several dark matter models have been proposed to fit best in radiuses. For example Kormendy and Freeman (2004) used pseudo-isothermal density profile that:

$$\rho_r = \rho_0 \frac{r_0^2}{r^2 + r_0^2} \quad (76)$$

Observations accompanied with this model result that it exist a constant central surface density of the dark matter $\rho_0 r_0$ and in this model this quantity takes a value of $\sim 100 \text{ Mpc}^{-2}$.

More recently, Spano *et al.* (2008) found a model that:

$$\rho_r = \rho_0 \left(1 + \left(r/r_0 \right)^2 \right)^{-3/2} \quad (77)$$

In this model it was resulted a constant central surface density of dark matter halo in an approximate value of 150 Mpc^{-2} .

Too newly studying a wide range of galaxies of different morphologies and with magnitudes in the interval $-22 < M_B < -8$ Donato *et al.* (2009) fit their rotation curves using a Burkert profile for dark matter (Burkert, 1995) that:

$$\rho_r = \frac{\rho_0 r_0^3}{(r + r_0)(r^2 + r_0^2)} \quad (78)$$

And it was found that:

$$\rho_0 r_0 \sim 140 \text{ Mpc}^{-2} \quad (79)$$

Then in these models there exist a constant central surface density of the dark matter as:

$$\rho_0 r_0 \approx \frac{1}{\pi} \quad (80)$$

And deviations are related to the shape of the profile.

In the fictitious dark matter model the calculations on the Newtonian gravity implies that:

$$\frac{GM_r}{r^2} = \frac{\Sigma_r}{\Sigma_r + \Xi_r} \frac{v_r^2}{r} \quad (81)$$

So that Ξ is fictitious dark matter surface density and Σ_r is observable surface density.

This shows that if the surface density of the dark matter be zero then the gravity is Newtonian but existence of non-zero density of the dark matter in the radii r causes that it be considerable a coefficient i that:

$$\frac{GM_r}{r^2} = i_r \frac{v_r^2}{r} \quad (82)$$

And by Equations (81, 82):

$$i_r = \frac{\Sigma_r}{\Sigma_r + \Xi_r} \quad (83)$$

In outer radii, the DM surface density is adequately larger than the baryonic mass surface density there, then in outer radii of the galaxies it is agreement that:

$$\frac{\Sigma_r}{\Xi_r} \gg 1 \quad (84)$$

Then from Equation (84) in outer radii it is good to write approximately that:

$$\Xi_r \approx i_r \Sigma_r \quad (85)$$

For a profile match with this dark matter surface density we should cut off the profile in lower radii. But the dark matter density in outer radii is larger than lower radii and then we can neglect the cut off in lower radii to consider a continuum best fit profile as Equation (85).

Then when in the central regions, the dark matter surface density is neglectable, for Newtonian core of the galaxies it is good to consider that:

$$i_0 = 1 \quad (86)$$

And substituting this relation into Equation (85):

$$\Xi_0 = \Sigma_0 \quad (87)$$

This result means that the pseudo central surface density of the galaxies is equal with baryonic central surface density of the galaxies in ideal disks.

Now because in ideal disks, the central surface density is considerable as a constant on the Freeman law (Equation 42) then from Equation (87) we deduce that:

$$\Xi_0 = \frac{1}{\pi} \quad (88)$$

Then when it is used density profiles similar to the Burkert profile it needs the halo have a central surface density ρ_0 that in a radii r_0 , the projected surface density of dark matter be:

$$\rho_0 r_0 \approx \Xi_0 \quad (89)$$

Then independency of the dark matter central surface density is directly dependent to the independency of the constant baryonic central surface density in Freeman law. This dark matter central surface density is not real suppose it is fictitious to fit best the dark matter profile with outer radiuses.

5. Milgrom Mond as an Empirical Non Fundamental Theory

5.1 From Machian Mond to Milgrom Mond in Regular Systems

Consider below formula for inertial potential at r as the arbitrary definition for a regular galaxy:

$$\psi_r = \frac{\sqrt{MM_{<r}}}{r} \quad (90)$$

The M is total mass and $M_{<r}$ is mass of the system below the radius r .

We use a scattering coefficient λ'' for this relation as:

$$\psi_r = \lambda'' \frac{\sqrt{MM_{<r}}}{r} \quad (91)$$

Substituting Equation (28) in the Equation (23) we obtain:

$$g^N = \frac{\psi_r}{\sqrt{2M}} a \quad (92)$$

Interestingly remembering a quotation by Julian Barbour (2001, p. 65) that:

"The (Newton's) law of inertia will turn out to be a motion relative to some average of all the masses in the universe."

Substituting Equation (91) in the Equation (92) we obtain:

$$g^N = \lambda'' \frac{a_r^2}{2G_N} \quad (93)$$

Milgrom's formula is extracted if the system be regular means $\lambda'' = 1$. Comparing this equation with Milgrom's formula we obtain that Milgrom has considered a fundamental constant acceleration a_0 in his theory instead $2G_N$ whereas we see that:

$$a_0 = 2G_N \quad (94)$$

Of course, the Equation (94) is agreement quantitatively with Milgrom phenomenological report for amplitude of his fundamental acceleration (Milgrom, 1983a; 1983b; 1983c).

And if we define a gravitational intensity g^M so that

$$g^M = a \quad (95)$$

Then by equation (93) for $\lambda'' = 1$, it is deduced that

$$g^M = \sqrt{2G_N g^N} \quad (96)$$

And this relation is Milgrom definition of his gravitational intensity g^M .

Then for a regular galaxy it is extracted below proposition:

$$\psi_r = \lambda'' \frac{\sqrt{MM_{<r}}}{r} \xrightarrow{MMOND} g_r = \lambda'' \frac{a_r^2}{2G_N} \quad (97)$$

This conditional proposition is a next verification for MMOND and showing that the Milgrom's MOND is an approximate empirical simulation specified in a case that the system is Sersician.

In reality the central surface density is constant ideally, where the Fishian relation is compatible whereas that while the central surface density is lower, then λ' too will be lower and lower λ' yields to a lower fictitious a_0 as reported by Swaters *et al.* (2010) that:

"We find that there appears to be a correlation between central surface brightness and the best-fit value of a_0 , in the sense that lower surface brightness galaxies tend to have lower a_0 ."

There are many papers reporting significant deviations from Milgromian constant acceleration not possible to cite

all here, for example a new result by Frandsen and Petersen (2018) or Ludlow *et al.* (2017) fit the Milgrom formula to their simulated galaxies, but they find a different value instead Milgromian acceleration.

5.2 Regular Systems are Sersician

Observationally it is visible that the galaxies and galaxy clusters are scattering around the point $\lambda'' = 1$. But for a pure argument we can use the Sersic profile which galaxies mainly are match. As a shell theorem, for inertial potential in a distance r in an isotropic disk galaxy (e.g. Fitzpatrick, 2012) "An introduction to celestial mechanics" we have:

$$\psi_r = \frac{M_{<r}}{r} + \int_r^R \frac{dm}{r} \quad (98)$$

And then to calculate λ'' we substitute ψ_r from Equation (91) into this Equation (98) we deduce:

$$M_{<r} + r \int_r^R \frac{dm}{r} = \lambda'' \sqrt{MM_{<r}} \quad (99)$$

Writing this equation in the surface density format results:

$$\int_0^r \Sigma_r r dr + r \int_r^R \Sigma_r dr + \lambda'' \sqrt{\int_0^R \Sigma_r r dr \times \int_0^r \Sigma_r r dr} \quad (100)$$

And for a galaxy with Sersic number $n = 1$ and scale length h it is resulted:

$$\begin{aligned} & h - (h+r)e^{-r/h} + r(e^{-r/h} - e^{-R/h}) \\ & = \lambda'' \sqrt{(h - (h+R)e^{-R/h})(h - (h+r)e^{-r/h})} \end{aligned} \quad (101)$$

This equation is invariant by units' transformation. We can transfer arbitrary the scale of the meter until length scale h to be at the order of unity. Then by Equation (101) we obtain:

$$\frac{1}{\lambda''} = \frac{\sqrt{1 - (1+r)e^{-r}}}{1 - e^{-r}} \quad (102)$$

Plotting Equation (102) in Fig. 1 is showing $\lambda'' = 1$. Actual density profiles better fits to $\lambda'' = 1$ for that Sersic profiles use from an extrapolated central surface density.

Then regular systems too are Sersician, that is, each galaxy matches the Sersic profile then its inertial potential obeys the Equation (91).

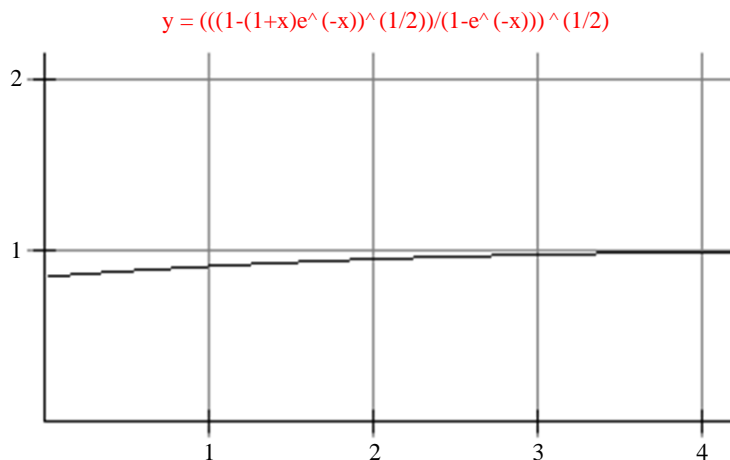


Fig. 1: Plotting $0.5 \lambda^{n-0.5}$ by vertical axis r in scale of $(h = 1)$

6 External Field Effect and quasi-Newtonian Gravity

By Equation (15), while the inertia of an external galaxy A is larger than internal inertia of a galaxy B , then in gravity equation of the galaxy B it requires to use the inertia in dominance of the external galaxy A . Then for a radii r of the galaxy B which the internal inertia is smaller than the inertia of the external galaxy A we have:

$$i_{int} > i_{ext} \Rightarrow (g)_B = i_A (a)_B \tag{103}$$

Then critical radii r_c of a galaxy is radii that the internal inertia is equal with inertia of the external galaxy. For higher radiuses, the gravity should be calculated in the inertia of the external galaxy. For a radii r of the galaxy B , beyond the critical radii with distance L from the external galaxy A , by Equations (103) and (92) we have:

$$(g)_B \frac{\sqrt{2M_A}}{\psi_{ext}} = (a)_B \tag{104}$$

And for external inertia we can write as a good approximation that:

$$\psi_{ext} = \frac{M_A}{L} \tag{105}$$

And by substituting this relation in the Equation (104) we have:

$$(g)_B \frac{\sqrt{2G_N}}{\sqrt{G_N M_A / L^2}} = (a)_B \tag{106}$$

And then mixing with Equation (96) we have

$$(g)_B \frac{2G_N}{g_{ext}} = (a)_B | g_{ext} = \sqrt{2G_N (g)_A} \tag{107}$$

This results quasi-Newtonian Gravitational G so that:

$$G = G_N \frac{2G_N}{g_{ext}} \tag{108}$$

On the other hand, by condition of external field effect (Equation 15) and Equations (11, 92, and 105) we obtain:

$$i = \frac{M_A / L}{\sqrt{2M_A}} = \frac{\sqrt{G_N M_A / L^2}}{\sqrt{2G_N}} = \frac{g^M}{2G} | g^M = \sqrt{2G_N g^N} \tag{109}$$

Then by Equations (15, 96, and 109), for external field effect it should be conditionally agreement that:

$$(g^M)_{ext} > (g^M)_{int} \tag{110}$$

And we have that:

$$i \leq 1 \tag{111}$$

Then we deduce that:

$$1 \geq i_{ext} \geq i_{int} \tag{112}$$

And mixing with Equation (109):

$$2G_N > (g^M)_{ext} > (g^M)_{int} \tag{113}$$

And this is a paradigm reported by Milgrom phenomenology (1983a; Bekenstein and Milgrom, 1984; Famaey and McGaugh, 2012) that:

“An external acceleration field, g_e , enters the internal dynamics of a system imbedded in it. For example, if the system’s intrinsic acceleration is smaller than g_e and both are smaller than a_0 , the internal dynamics is quasi-Newtonian with an effective gravitational constant Ga_0/g_e (Milgrom, 1983a; 1986b; Bekenstein and Milgrom, 1984). This was applied to various astrophysical systems such as dwarf spheroidal galaxies in the field of a mother galaxy, warp induction by a companion, escape speed from a galaxy, departure from asymptotic flatness of the rotation curve and others.”

7 Newtonian Regimes of High Surface Brightness Galaxies and Relevant Paradigms

We have an evident proposition that:

$$r_a < r_b \Rightarrow \phi_a > \phi_b \quad (114)$$

This proposition means that in lower radiuses, the inertial potential is larger.

If there is a maximum size of transition radius r_t for agreement of below equality that:

$$\phi_{r_t} = \phi_0 \quad (115)$$

Then because the central inertial potential is the maximum, it will be resulted by Equations (114, 115) that:

$$r < r_t \Rightarrow \phi_r \cong \phi_0 \quad (116)$$

Then according to Equations (22, 116) it is resulted:

$$r < r_t \Rightarrow g_r^N = a_r \quad (117)$$

And then if there is such a radius r_t conditioned in Equation (115), we have a Newtonian regime for $r < r_t$.

Is there such a radius in the galaxies which the inertial potential in that radius to be equal with central inertial potential? To answer we need to embed the Equations (29, 90) into the Equation (115) as:

$$\frac{\sqrt{MM_{<r}}}{r} = \sqrt{2M} \quad (118)$$

$$a_{r_t} = 2G_N \quad (119)$$

This is showing that the transition radius of a Newtonian regime is at the boundary with acceleration

equal to $2G_N$ verifying next phenomenological paradigms of the Milgrom that:

- “In a disc galaxy, whose rotation curve is v_r , that has high central accelerations ($v_r^2/r > a_0$ in the inner regions), the mass discrepancy appears always around the radius where $v_r^2/r = a_0$. In galaxies whose central acceleration is below a_0 (low surface brightness galaxies—LSBs) there should appear a discrepancy at all radii (Milgrom, 1983b).”

Of course λ' and λ'' are not actually exact on the order of unity because that the galaxies are scattering around the ideal Sersician systems and then transition radiuses of the galaxies actually are scattering around a mean value at acceleration equal to $2G_N$. This phenomenon has been reported before as one of the Milgrom paradigms that:

- “For a concentrated mass, M , well within its transition radius, $r_t = \sqrt{MG/a_0}$, r_t plays a special role (somewhat akin to that of the Schwarzschild radius in General Relativity) since the dynamics changes its behaviour as we cross from smaller to larger radii. For example, a shell of phantom DM may appear around this radius (Milgrom and Sanders, 2008).”

Also in the paper (McGaugh *et al.*, 2016), McGaugh and Lelli and Schomberg analysed data from a set of about 150 disk galaxies in the prime. They identified the best-fitting acceleration scale for each of them and found that the distribution is clearly scattering around a mean value about $2G_N$. Of course on the same data, a new report (Rodrigues *et al.*, 2018) is showing a monotonically deviation from Milgromian constant which not possible to illustrate aligned with mean value of the Milgrom. Probably, the reason of discrepancy is technical, on the different styles considered in the analysis, means Gaussian priors by McGaugh *et al.* instead flat priors over a finite bin.

Also in MMOND it is clear that:

$$r > r_t \Rightarrow a_r < 2G_N \quad (120)$$

This shows that Milgromian deep MOND appears where, $a < 2G_N$ in agreement with phenomenological reports without any type of cosmological hypothesis despite the Milgrom's hypothesis about a fundamental constant acceleration a_0 .

Also for Newtonian regime it is extrapolated from Equations (29, 90 and 115) that:

$$r \leq r_t \Rightarrow \frac{\sqrt{MM_r}}{r} \cong \sqrt{2M} \quad (121)$$

And then it is deduced that:

$$r \leq r_t \Rightarrow a_r \cong 2G_N \quad (122)$$

Then in the Newtonian core of the HSB galaxies too, the acceleration will not much exceed than $2G_N$. This phenomenon has been reported too as one of the Milgrom's paradigms that:

- "The excess acceleration that MOND produces over Newtonian dynamics, for a given mass distribution, cannot much exceed a_0 (Brada and Milgrom, 1998)."

The surface density and acceleration are at the same dimension and then this paradigm is possible to translate to the surface density format as noted by Milgrom that:

- "Isothermal spheres have mean surface densities $\bar{\Sigma} \leq \Sigma_0 = a_0 / \pi G$ (Milgrom, 1984) underlying the observed Fish law for quasi-isothermal stellar systems such as elliptical galaxies."

Simply derivable from correlation of the gravity and mean surface density within transition radius so that:

$$g_{r_t}^N = 2G_N \quad (123)$$

And then:

$$\frac{M_{r_t}}{r_t^2} = \frac{2}{\pi} \quad (124)$$

Then critical mean surface density at transition radii is:

$$\Sigma_{r_t} = 2 / \pi \quad (125)$$

And this is generalizable to the mean surface density by Equation (117). Finally we understand that against the Milgrom theory, there is no a fundamental constant acceleration a_0 in physics, on the contrary Newtonian regime of the galaxies is duo to natural constancy of the inertia i in that areas.

8 Galaxies Clusters and Globular Clusters Difficulties with Milgrom's MOND

8.1 The Discrepancy by External and Internal Field Effects and Tully-Fisher Relation Discrepancy in Context of the Observable Matter

It was realized early on (The and White, 1988; Gerbal *et al.*, 1992; Sanders, 1994; 1999; 2003;

Aguirre *et al.*, 2001; Pointecouteau and Silk, 2005; Takahashi and Chiba, 2007; Angus *et al.*, 2008; Milgrom, 2008; Hodson and Zhao, 2017) that the Milgrom's MOND does not fully explain away the mass discrepancy in galaxy clusters. About Globular clusters too we have difficulties (Baumgardt, 2006; Ibata *et al.*, 2011) included to remarked deviations from Milgrom's MOND.

According to Equation (3), the discrepancy between observations and Milgrom's MOND is the same discrepancy between observations and Tully-Fisher relation. Tully-Fisher relation does deviate from its standard format from galaxies to galaxy clusters (McGaugh, 2007) as noted by Stacy McGaugh that:

"Extrapolation of the Baryonic Tully-Fisher relation to cluster scales suggests that the inventory of baryons in clusters may be incomplete."

Milgrom (2008) alternative is that this matter is baryonic in some yet unidentified form, such as cold, dense clouds. Another assumption, it is massive neutrinos (Sanders, 2003; Angus *et al.*, 2008). Bekenstein (2011) proposed that the Milgrom constant a_0 is a monotonically increasing function of potential depth Which boosts the Milgrom MOND in galaxy clusters by a factor of a few, and Zhao and Famaey (2012) have proposed EMOND to resolve the puzzle of clusters by handy effective changes in the Migrom MOND as a generalized Milgrom constant $A_0 \sim a_0(\varphi^2 / \varphi_0^2)$ and such a way has been continued by Hodson and Zhao (2017) and Khoury has considered GMOND (Khoury, 2015). But all these models are mathematical simulations.

The observable matter density in the galaxies and cluster of the galaxies is very larger than that of the universe mean density and then deviation from Tully-Fisher relation can't be related to the dark matter but it should be resolved by observable matter. As it is visible in Figure 2, the Tully-Fisher normalization factor k ($v_\infty^4 = kM$) is larger in larger systems till flattening to a maximum size. The surface density of the galaxy clusters follow the Sersic profiles and then in usual conditions, it should be agreement $\lambda' = 1$, that is, standard Tully-Fisher relation. But the quantum bound systems have sub quantum bound systems inside, similar to the globular clusters inside the host galaxy. Then the mass under dominance of the dwarf galaxies and globular clusters inside a host galaxy or galaxies in the galaxies clusters strongly would not be used in the inertia integration for host galaxy or host cluster. This effect is named here internal field dominance (IFD).

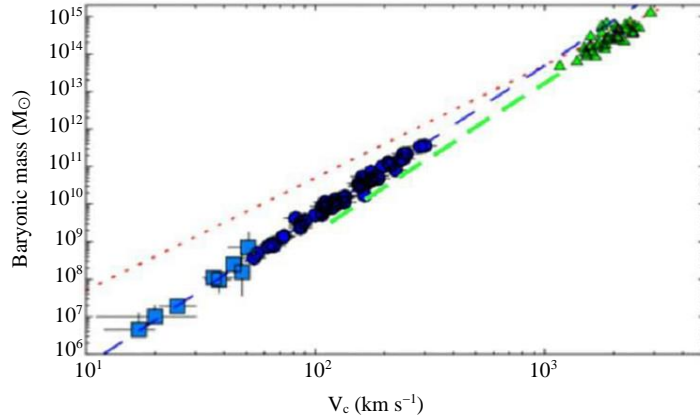


Fig. 2: The baryonic mass-circular velocity relation, spanning the range from the tiniest dwarfs (squares) through spiral galaxies (circles; [Mcgaugh (2005)]) to cluster of galaxies (triangles; [Reiprich (2001)]). The dashed blue line is the fit to the galaxy data. The dotted line is what the scientists expect in Λ CDM??

When there is external field dominance on a system which we want to calculate gravity in a point of that, for example a galaxy under external field of a host galaxy, then according to Equation (15) it needs to discount the mass under dominance of the external field from the inertia integration in that point. This quantum effect exists also for internal field dominance of the sub quantum bound systems, inside a host galaxy or galaxy cluster.

Then Equation (23) under external and internal field dominance is written as follows:

$$g^N = \frac{\sum_{k=1}^N \frac{m_k}{r_{kp}} - \sum_{k \in \{EFD, IFD\}} \frac{m_k}{r_{kp}}}{\sum_{k=1}^N \frac{m_k}{r_{ko}} - \sum_{k \in \{EFD, IFD\}} \frac{m_k}{r_{ko}}} a \quad (126)$$

For accurate calculations, this equation should be used. The matter under external field dominance is generally tiny compared to the full inertial potential and then we can use the Equation (92) under IFD so that:

$$g^N = \frac{\sum_{k=1}^N \frac{m_k}{r_{kp}} - \sum_{k \in \{IFD\}} \frac{m_k}{r_{kp}}}{\sqrt{2M}} a \quad (127)$$

Assuming a regular system, that is $\lambda'' = 1$, it is deduced:

$$2G_N^2 M_{<r} = \left(1 - \frac{\phi_{IFD}}{\phi_r}\right)^2 v_r^4 \quad (128)$$

Then it is deduced that:

$$g_r^N = \left(1 - \frac{\phi_{IFD}}{\phi_r}\right)^2 \frac{a_r^2}{2G_N} \quad (129)$$

This relation is showing a generalized form of Milgrom formula and then Milgrom fictitious constant acceleration a_0 is generalized as:

$$A_0 = \frac{2G_N}{\left(1 - \frac{\phi_{IFD}}{\phi_r}\right)^2} \quad (130)$$

And also from the Equation (128) it is deduced a generalized Tully-Fisher relation as:

$$v_\infty^4 = \frac{2G_N}{\left(1 - \frac{\phi_{IFD}}{\phi_r}\right)^2} G_N M \quad (131)$$

We can see a correlation between generalized Tully-Fisher relation and generalized Milgrom constant as a very strong verification for MMOND. Of course the galaxies \mathcal{L} should be accounted in the calculations. For the systems included to sub systems, the percentage of inertial potential in dominance of the internal field to the inertial potential in dominance of the host galaxy or cluster increases up to a maximum percentage because that the internal field is not actually dominant on the main of the inertial potential in a galaxy or cluster. This means that it is actually or even mathematically impossible that ϕ_{IFD} to be equal to ϕ_r of host galaxy or host cluster and then there should be the maximum value less than unity for ϕ_{IFD}/ϕ_r . It is almost rare to find a galaxy cluster with $\phi_{IFD}/\phi_r > 2/3$. Without such a

maximum for this proportion it would appear serious problems like the problem of Bekenstein formula as noted by Zhao and Famaey (2012) that:

“However, a side effect of Bekenstein’s exponentially-varying function is that it predicts a value of a_0 of the order of ms^{-2} once the potential reaches the order of c^2 , i.e., a neutron star or a stellar black hole would exhibit an undesirable deep-MOND behavior.”

The maximum amplitude of A_0 has been detected observationally as discussed in (Zhao and Famaey, 2012; Hodson and Zhao, 2017). By the way, Tully-Fisher relation normalization factor is limited to a maximum A_0 as $A_{0,m}$. If we assume this maximum, where ϕ_{FD} is 2/3 of the potential ϕ_r then according to Equation (130), we deduce that:

$$\frac{\phi_{FD}}{\phi_r} \leq \frac{2}{3} \Rightarrow A_{0,m} = 9a_0 \quad (132)$$

Then Tully-Fisher relation has a transition from standard Tully-Fisher in low scale to its very large scale version (M in sun's mass and v in 1 km/s) as:

$$\log M = 1.75 + 4 \log v_\infty \rightarrow \log M = 0.8 + 4 \log v_\infty \quad (133)$$

This relation is showing the jump in the normalization factor of the standard Tully-Fisher relation to a larger constant which is visible in the reports for galaxies clusters. In Fig 2. It is visible that the line of Tully-Fisher relation for galaxies clusters is parallel with line of the standard Tully-Fisher relation for galaxies, verifying Equation (133).

Stars and the planets have strong gravitational accelerations. Then it may be assumed that these masses should be quantum bound systems by large internal fields. But despite the Milgrom's theory, the boundary of gravitational systems generally is not by acceleration on the contrary by inertia. Referring to the Equation (11) and (22) we find the inertia i as:

$$i = \frac{\phi_r}{\phi_0} \quad (134)$$

By this relation we can see that the inertia i for all planets and stars are lesser than the order of unity ($i < 1$). For example, on the surface of the Earth, the inertia i is calculated about 1/3 whereas that the inertia i for galaxies is about the order of unity. Then the stars and planets are not quantum bound systems and they should be accounted in inertia integration of their host galaxy or their host cluster.

8.2 Rich Globular Cluster Galaxies (UDGS: Ultra Diffuse Galaxies) and Missing Dark Matter

The most widely reported discovery of a galaxy that seemed to lack dark matter came in March 2018. A team of astrophysicists led by (van Dokkum *et al.*, 2018) showed that the average speed of globular clusters in galaxy NGC 1052–DF2 matched a baryons-only galaxy model as said (Live Science) lead author Pieter van Dokkum of Yale University that:

“Finding a galaxy without dark matter is unexpected because this invisible, mysterious substance is the most dominant aspect of any galaxy,”

And too as said by Pieter Van Dokkum that:

“For decades, we thought that galaxies start their lives as blobs of dark matter. After that everything else happens: gas falls into the dark matter halos, the gas turns into stars, they slowly build up, then you end up with galaxies like the Milky Way. NGC1052-DF2 challenges the standard ideas of how we think galaxies form.”

The ultra-diffuse galaxy is rich with globular clusters, which hold the key to understanding this mysterious object’s origin and mass. A closer look at one of the globular clusters within the galaxy, which are all much brighter than typically seen, with the brightest emitting almost as much light as the brightest globular cluster within the Milky Way. The spectrum, obtained by Keck Observatory, shows the calcium absorption lines used to determine the velocity of this object. 10 clusters were observed, providing the information needed to determine the mass of the galaxy, revealing its lack of dark matter.

The research, published in the March 29th issue of the journal Nature, amassed data from Gemini North and W. M. Keck Observatory, both on Maunakea, Hawaii, the Hubble Space Telescope and other telescopes around the world. Given its large size and faint appearance, astronomers classify NGC1052-DF2 as an ultra-diffuse galaxy, a relatively new type of galaxy that was first discovered in 2015.

Van Dokkum explained that “If there is any dark matter at all, it’s very little.” “The stars in the galaxy can account for all of the mass and there doesn’t seem to be any room for dark matter.”

A second galaxy missing dark matter in NGC 1052 is reported again van Dokkum *et al.* (2019) supporting the existence of null dark matter galaxies.

The find dramatically increases the number of galaxies that appear to be missing dark matter. Researchers have found that certain small galaxies, now

including these 19, behave as if they're dominated by baryons - the particles that make up ordinary matter. The newest paper, published Nov. 25 in the journal Nature Astronomy (Guo *et al.*, 2020), identified the 19 dark matter-free galaxies using the same method as a newly published news in www.space.com by Rafi Letzter December 05, 2019:

"19 Galaxies Are Apparently Missing Dark Matter. No One Knows Why."

But here we have answered it explicitly on the evidence that the globular clusters have Internal Field Dominance (IFD) reasonable to discount a noticeable size of the inertial potential inside the host galaxy as following by Equation (126) that:

$$g^N = \frac{\sum_{k=1}^N \frac{m_k}{r_{kp}} - \sum_{k \in \{IFD\}} \frac{m_k}{r_{kp}}}{\sum_{k=1}^N \frac{m_k}{r_{ko}} - \sum_{k \in \{IFD\}} \frac{m_k}{r_{ko}}} a \quad (135)$$

Then for two similar mass profile galaxies, it may be deduced different rotation curves for that the gravitational potential is active but the inertial potential is passive so that the mass under dominance of the sub quantum bound systems inside a host galaxy is discounted from the inertial potential integration. Then the globular clusters as the sub quantum bound systems cause to appear hollow in the inertial potential in the gravity equation of a body in the inside of rich globular cluster host galaxy. The sub system is being an independent system and the matter in the sub system is rotating around the rest of the sub system. The globular clusters are distributed mainly in the central region of their host galaxy and then to cause a hollow for inertial potential in central region of the galaxies included to the globular clusters inside. Then according to the potential shell theorem (e.g. Fitzpatrick, 2012), for inertial hollow inside of an isotropic shell, for rich globular cluster galaxies we have:

$$\sum_{k=1}^N \frac{m_k}{r_{kp}} \cong \sum_{k=1}^N \frac{m_k}{r_{ko}} \quad (136)$$

Substituting this relation into Machian NOND (Equation 22) yields to a Newtonian gravity. Then inertial hollow generated by globular clusters yields the galaxy to the side of the Newtonian galaxy in the context of observable matter. Then the missing fictitious dark matter in the UDGs is result of the quantum bound systems and their internal field dominance in MMOND. The phenomenon in the UDGs is inverse of the phenomenon in the galaxy clusters because that in the

galaxy clusters, the distribution of the galaxies as the sub quantum bound systems in the inside of the host cluster is rich the more outside, but in the UDGs, the globular clusters are rich in the center. Then in the galaxy clusters, the central inertial potential is not varied noticeably by IFD whereas in outer regions, the inertial potential is varied noticeable relatively.

In the UDGs, for almost equality of the central inertial potential with inertial potential in the out to the outermost measured points contrary to the other galaxies, the gravity equation is not match with Milgrom formula because that Milgrom formula is an approximate relation agreement empirically when the galaxy is ideally Fishian and Fishian galaxies are following below equations that:

$$\begin{cases} \psi_r = \sqrt{MM_{<R}} / r \\ \psi_0 = \sqrt{2M} \end{cases} \quad (137)$$

But both of these conditions are invalid in UDGs for existing sub quantum bound systems and then Milgrom MOND never predicts Newtonian galaxy in LSB and Milgrom MOND is not valid in UDGs too. For equivalency of the Milgrom empirical formula and Tully-Fisher relation, the UDGs outlier the baryonic Tully-Fisher relation. This phenomenon has been reported newly by a group of scientists (Pina *et al.*, 2019) in title "off the baryonic Tully-Fisher relation: A population of Baryon-dominated Ultra-diffuse galaxies".

9 Bullet Clusters and Multi Nuclei Galaxies Difficulties with Milgrom's Mond

For bullet galaxies or multi-nuclei galaxies we can see that these galaxies are irregular $\lambda'' \neq 1$ and non Fishian $\lambda' \neq 1$ and the Milgrom MOND is not match in these galaxies as we see high discrepancies for bullet cluster 1E0657-56 (Clowe *et al.*, 2006), Or in central regions of a type of galaxies. For example, as a new paper by Israa *et al.* (2018), in central region of NGC 3256 which the gravity should be Newtonian in context of the Milgrom MOND, we have a high discrepancy. For NGC 3256 we use from Equation (22) that:

$$g_r^N = \frac{\phi_r}{\phi_0} a_r \quad (138)$$

By shell theorem for inertial potential we have for radius $r = 9.2$ arcsec of the NGC 3256 that:

$$g_r^N = \frac{M_{<r}}{r} + \frac{\int_r^{9.2} dm}{r} a_r \quad (139)$$

On the observable mass profile in (Israa *et al.*, 2018) we consider the radius 0.8 arcsec as a minimum radius which the mass is readable in confidence so that:

$$\psi_0 = \int_0^{9.2} \frac{dm}{r} = \int_0^{0.8} \frac{dm}{r} + \int_{0.8}^{9.2} \frac{dm}{r} \quad (140)$$

And referring to paper by Israa *et al.* (2018), we deduce:

$$m = 1.87 \times 10^{20} r + 3.21 \times 10^{39} \quad (0.8 \text{ arcsec}) \leq r \leq (9.2 \text{ arcsec}) \quad (141)$$

By observable matter data from (Israa *et al.*, 2018), however there is uncertainty about the light to mass ratio below the 0.8 arcsec, it is deduced that

$$\psi_0 \sim 2 \times 10^{21} \quad (142)$$

In reality we need to modify a bit the mass profile drawn in (Israa *et al.*, 2018), as reformed by green line in the Fig. 3 for inflation of light L. NGC 3256 is not just a galaxy but included to the two colliding galaxies. Then we need to consider high amplitudes of the frictional radiation deduced by collision of these galaxies. This effect increases highly the light to mass ratio in the dense areas of the NGC 3256 as a source for inflation of visible light below the 0.8 arcsec.

NGC 3256 is not a galaxy but two combined galaxy colliding together reasonable for a negative value of fictitious dark matter for radiuses lower than the distance

between their nuclei because that in such a colliding galaxies, the center of mass is not exactly at the same point that the inertial potential is maximum and such a negative density of fictitious dark matter in the regions between the nearby galaxies has been reported by Milgrom (1986b). But this negative dark matter is neglect able here.

Then for $r < 0.8 \text{ parsec}$ for NGC 3256 it is suitable to use from mass profile deduced by Newtonian gravity. This assumption results:

$$r \leq 0.8 \text{ arc sec} \Rightarrow m = 200r^2 \quad (143)$$

Then by Equations (140, 141, 143) we obtain:

$$\psi_0 \sim 2.16 \times 10^{21} \quad (144)$$

Then calculation of the central inertial potential by reformed mass profile for $r < 0.8 \text{ arc sec}$ is too fit to the amplitude visible by (Israa *et al.*, 2018). Then this size of central inertial potential is on the confidence, whether we use Newtonian gravity for $r < 0.8 \text{ arc sec}$ or using observable matter by (Israa *et al.*, 2018).

By the way we obtain for NGC 3256 that:

$$\lambda' = \frac{\psi_0}{\sqrt{2M}} = \frac{2.16 \times 10^{21}}{\sqrt{2 \times 1.6 \times 10^{40}}} = 12 \quad (145)$$

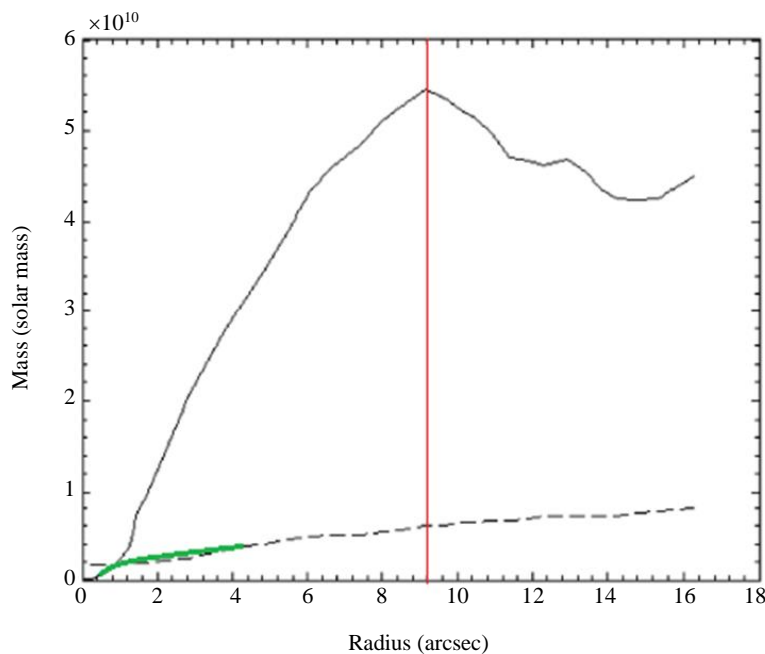


Fig. 3: Total mass and baryonic mass of NGC 3256 as a function of radius by Israa *et al.* The solid line is the total mass of NGC 3256 and the dashed line is the baryonic component of NGC3256 and very sharp break of the gravity gradient at $r = 9.2 \text{ arcsec}$

And then NGC 3256 is a highly fake galaxy in context of the observable matter. The Equation (139) is written also as:

$$g_r^N = \frac{\frac{M_{<r} + \int_r^{9.2} dm}{r}}{2.16 \times 10^{21}} a_r \quad (146)$$

Substituting Equation (143) in this equation results (r in arcsec) that:

$$g_r^N = \left(0.086 + \frac{0.291}{r} + 0.086 \ln \left(\frac{9.2}{r} \right) \right) \times a_r \quad (147)$$

This is carefully agreement with reported accelerations by Israa *et al.* (2018) and also we see that for NGC 3256 in radius 1.7 kpc we observe $a = 8G_N$ versus the Milgrom's MOND which predicts Newtonian gravity.

On the other hand, from mass profile (Israa *et al.*, 2018) in Fig. 3, we see a very sharp break for the gravity gradient at the point 9.2 in arcsec, changing sharply to a quasi-Newtonian regime for radiuses rather than 9.2 arcsec. Quasi-Newtonian type gravity is visible since we have an EFE on a host galaxy (Milgrom, 1983a; 1986a; Bekenstein and Milgrom, 1984). Then from Equation (15), the internal inertia and external inertia at 9.2 arcsec should be equal as critical condition of external field effect in MMOND. From Equation (134) we have that:

$$i_{ext} = \frac{M_{<9.2} / 9.2 \text{ arc sec}}{\psi_0} = 0.12 \quad (148)$$

This external filed inertia should be relevant to a nearby galaxy around the NGC 3256. If we assume such a galaxy with mass M' and distance L' from NGC 3256, then from Equations (93) and (149), for equality of external and internal inertia in NGC 3256 we have:

$$i_{ext} = \frac{M' / L'}{\sqrt{2M'}} = \sqrt{\frac{M'}{2L'^2}} = 0.12 \quad (149)$$

$$g_{ext}^N = 2 \times 10^{-12} \quad (150)$$

In reality all galaxies in the universe have smaller effect on the NGC3256 and greatest effect is just relevant to the nearby galaxy NGC 3256C. It is wonderful that NGC 3256C has the amplitude of g over the NGC 3256 equal to 2×10^{-12} . Then for NGC 3256, in a similar mathematical manner used in Equation (107) it is deduced:

$$r \geq 9.2 \Rightarrow g_r^N = \frac{\sqrt{2G_N g_{(NGC-3256C)}^N}}{2G_N} a_r \quad (151)$$

And this equation shows a quasi-Newtonian gravity which causes to decrease rapidly the acceleration for radiuses larger than 9.2 arcsec in NGC 3256, compatible with rotation curve detected by Israa *et al.* (2018). But in Milgrom MOND (Milgrom, 1983a), it is impossible such an EFE because that at radius 9.2 arcsec, Milgrom MOND shows $a_{r=9.2} = 2.8 \times 2G_N$ and this incompatibility with Milgrom MOND is related to the fact that the NGC 3256 is highly non Sersician and too relevant to the reality that the condition of external field effect in Milgrom MOND is not correct generally and Milgrom MOND is working as an approximation, where the system is consistent to the Tully-Fisher relation. When the galaxies are colliding, the resultant galaxy becomes a highly fake galaxy reasonable for high discrepancy in the colliding galaxies.

10 Modified Gravity in the Solar System

From Equation (22), the gravity is Newtonian-type with variable G limited to the boundary of the Milky Way galaxy so that:

$$G = G_N \frac{\psi_0}{\psi_r} \quad (152)$$

Then by Equation (29), this equation is written as follows:

$$G = G_N \frac{\lambda' \sqrt{2M}}{\sum_{k=1}^N \frac{m_k}{r_{kp}}} \quad (153)$$

M is total mass of the Milky Way galaxy and n should be the number of the particles m_k limited to the boundary of the Milky way galaxy. Substituting Equation (90) in the Equation (153) we obtain:

$$G = G_N \times \frac{\lambda' r}{\sqrt{M_r / 2}} \quad (154)$$

In the solar system, the gravity is Newtonian and then it should be agreement that:

$$\frac{\lambda' R}{\sqrt{M_R / 2}} = 1 \quad (155)$$

So that R is distance of the Earth from center of the Milky Way galaxy and $M_{<R}$ is mass of Milky Way for $r < R$. Assuming Milky Way as a Sersician galaxy we obtain by Equation (155) that:

$$g_R = 2G_N \quad (156)$$

This equation is showing that the solar system should be on the Newtonian regime of the Milky Way galaxy and this is a reality, verifying modified Newton gravity by Mach's inertia principle.

Then Newton gravity in the solar system is modified as:

$$F = \frac{G_N R}{\sqrt{M_{<R}/2}} \times \frac{mm'}{r^2} \quad (157)$$

Amazingly we see that the gravity at the Earth position is relevant to the mass of the Milky Way galaxy and position of the Earth in the Milky Way.

11 The Yukawa Strong Force is Finite-Range Gravity

In Mach's mechanics, the force is not action at a distance but it needs force carrier and then the gravity is finite range. Such a mechanism has been examined initially in (Freund *et al.*, 1969) and continued in more detail by many scientists not possible to mention all here, as newly discussed by Valev (2015) a finite-range Yukawa type gravity at the size of Hubble radius for assumption that the graviton range is equal to the radius of observable universe as noted by Valev (2015) that:

"The presence of an exceptionally small, yet non zero mass of the graviton, involves a finite range of the gravity, $r_g \sim \lambda_g$ and Yukawa potential of the gravitational field $\varphi_r = \frac{G_N M}{r} \exp\{-r/\lambda_g\}$ where λ_g is Compton wavelength of the graviton $\lambda_g = h/m_g c = c/H$ "

Since Compton wavelength is at the order of Hubble radius, the mass of theoretical graviton is deduced at the order of hH/c^2 as we have:

$$m_g c = \frac{h}{\lambda_g} \Rightarrow m_g = \frac{hH}{c^2} \quad (158)$$

This mass coincides minimum quantum of energy from Alfonso-Faus (2012; Alfonso-Faus and Alfonso, 2013). This minimum quantum of energy seems close to the graviton mass obtained by different methods (Woodward *et al.*, 1975; Gershtein *et al.*, 1997; Valev, 2005; Alves *et al.*, 2011). Also this quantity takes substantial places in the estimations of total information and entropy of the universe (Gkigkitzis *et al.*, 2013; Haranas and Gkigkitzis, 2013). This value of graviton mass also has been found (Valev, 2013; 2017) by fundamental constants only, without consideration of any cosmological models.

In fact the gravity is carried by all particles as noted by Dimitar Valev that:

"All particles and masses take part in gravitational interaction so they emit and absorb virtual massive gravitons and nucleons in the nuclei emit and absorb virtual massive pions. So the galactic clusters interact gravitationally via particles building them."

But in the scale of atomic nuclei, the gravity is carried mainly by heavy particles or so called hadrons and then the gravitational drop would be happened in the range, the hadrons carry the gravity.

By the way, the gravity carrier in microscopic range is mainly the hadron whereas the theoretical gravitons are mainly gravity carrier in large scale. Then the range of gravity since the hadrons are gravity carrier would change. We can calculate it mathematically on the base of Compton wavelength proportionally as:

$$\frac{m_{Hadron}}{m_g} = \frac{\lambda_g}{\lambda_{Hadron}} \quad (159)$$

Then the range of gravity, carried by hadron is not the same by graviton. On the results (e.g. Freund *et al.*, 1969), the range of graviton is Hubble radius R_H and then we obtain:

$$\lambda_{Hadron} = \frac{R_H m_g}{m_{Hadron}} \quad (160)$$

Substituting $m_g = hH/c^2$ (Smallest mass) in the Equation (160) results:

$$\lambda_{Nucleon} = 1.3214 \times 10^{-15} m \quad (161)$$

Then the nucleuses are gravitational micro finite-range systems which the gravity mainly is carried by hadrons. If $V(g)$ is the gravitational potential which is carried by gravity carrier then in comparison to the screened coulomb potential, it should be attached an exponential wave function in the format of Yukawa potential $V(g_{Yukawa})$ as:

$$V(g_{Yukawa}) = V(g) e^{-r/\lambda} \quad (162)$$

This exponential coupling effect is the result of the wave mechanism of force carriers and result of Schrodinger wave equation which initially was discovered by Yukawa (1935).

Now we need to know the gravitational potential $V(g)$. Nucleuses are not bounded inertial unlike the galaxies, but they are quantum mechanical finite-range

systems. Then their gravitational degree of freedom should be defined on the Machian relation as the gravity equation represented in the Equation (7).

According to Equation (7), for an arbitrary passive nucleon with mass M_{Nu} in a nucleus the gravity is:

$$\frac{M_{Nu}c^2}{r^2 \sum_{k=1}^N \frac{m_k}{r_{kp}}} = a \quad (163)$$

Consider a simple model of nucleus which we want to calculate the gravity between two particles in a distance r , since all other nucleons are simply assumed around the passive nucleon in the range of gravity. This is simply nNF system (n nucleons nuclear force). Gravitational potential for passive nucleon is calculated by integration on $n-1$ nucleons m_i/r_i and self-potential of the passive nucleon. The self-potential of the passive nucleon is calculated in the same manner in the book "MOED". (Lutephy, 2016). In this book we see that the difference between self-electric potential of the proton and electron results fundamental charge asymmetry. Here too it must be loaded in the equations, a self-gravity depended potential.

To calculate the self-gravity potential of the proton in its center we use a simple model of a sphere with radius of proton included to three spheres as the three quarks. In this model, the self-potential of the proton in its center is almost equal to:

$$\sum_{self} \frac{m}{r} \cong 3 \frac{m_{quark}}{0.5r_p} = 2 \frac{m_p}{r_p} \quad (164)$$

The m_p is mass of the proton and r_p is radius of proton. Of course in the section 12 we argue that this relation (164) is exact, that is $\sum_{self-proton} \frac{m}{r} = 2 \frac{m_p}{r_p}$.

Since there are $n-1$ number of nucleons around the passive nucleon in the range of gravity carrier, then for approximate equality of proton and neutron masses it is deduced by Equation (163, 164) that:

$$g = \frac{M_{Nu}c^2}{r^2 \left(2 \frac{m_p}{r_p} + \sum_{k=1}^{N-1} \frac{m_p}{r_{kp}} \right)} = a \quad (165)$$

We need to notice that n is not the number of nucleons in a nucleus suppose n is nearby nucleons in the range of gravity. According to the Equations (162, 165) we have a Machian gravitational potential $V(g)$ in nucleus as follows:

$$V(g_{Yukawa}) = \frac{M_{Nu}c^2 r_p}{m_p r \left(2 + \frac{r_p}{r} + r_p \sum_{k=1}^{N-2} \frac{1}{r_{kp}} \right)} e^{-r/\lambda} \quad (166)$$

This is the same origin of nuclear strong force in nNF mode.

We see that mutual interaction of the nucleons becomes weaker by increase of number of nucleons in the range in agreement with experimental reports by scientists, that is, so called saturation properties of the nuclear force (etc., Feenberg, 1937; Day, 1983).

In fact the saturation properties of the nuclear force which was a mystery is resolved ultimately here. The volume of nucleus is proportional to the number n of the nucleons and then it is increased the number of nucleons in each volume with radius equal to the range of strong gravity. Then for larger nuclei the strong gravity is weaker and this effect too causes naturally decrement of the range of strong force.

In large n , the n dependency of potential is noticeable as noted by (Critchfield and Teller, 1938) "On the Saturation of Nuclear Forces" that:

"When many heavy particles interact the total potential energy is found to be proportional to the number of heavy particles".

We find that When n is increased in the range of strong force, the mutual gravitational interaction between nucleons becomes weaker because that the nucleons potential prevails to self-potential of passive nucleon. Then when the active and passive nucleons are closest in comparison to other nucleons or for 2NF system, for equality of proton and neutron masses, from Equation (166) we obtain:

$$V(g_Y) = \frac{c^2 r_p}{r \left(2 + \frac{r_p}{r} \right)} e^{-r/\lambda} \quad (167)$$

The radius of nucleon is ever smaller than the distance r and then approximately we have:

$$V(g_Y) = \frac{c^2 r_p}{2r} e^{-r/\lambda} \quad (168)$$

This is Yukawa force, resolved here on a meaningful base.

Interestingly we see that 2NF nuclear force, depends alone to the light speed and radius of nucleon and the wavelength of the force carrier.

Comparison to newton's gravity yields to a strong gravitational G_Y as follows:

$$G_Y = \frac{c^2 r_p}{2m_p} \quad (169)$$

Now we compare the gravitational strong force with electrostatic force here. We compare these forces at the radius $r = 1$ fm. 1 fm is standard assumption used by scientists for comparison of nuclear strong force with electrostatic force.

First we calculate the g_Y by Equation (167) so that:

$$g_Y = \frac{c^2 r_p}{r^2 \left(2 + \frac{r_p}{r}\right)} \times \left(1 + \frac{r}{\lambda}\right) e^{-r/\lambda} \quad (170)$$

Comparison of g_Y and E at 1fm results:

$$\alpha = \frac{g_Y}{E} = \frac{m_p c^2}{k e^2} \times \frac{r_p}{\left(2 + \frac{r_p}{1\text{fm}}\right)} \times \left(1 + \frac{r}{\lambda}\right) e^{-r/\lambda} \quad (171)$$

Gravitational G and light speed c and proton's mass m_p and electric fundamental charge e and coulomb constant k are well established values. Substituting these values in the Equation (171) and assuming λ equal to the Compton wavelength of nucleons yields to the equation below:

$$\alpha = 537.5 \times \frac{(r_p / 1\text{fm})}{2 + (r_p / 1\text{fm})} \quad (172)$$

Historically the proton radius was measured by two independent methods, which converged to a value of about 0.877 femtometres. This value was challenged by a 2010 experiment using a third method, which produced a radius about 4% smaller than this (Pohl *et al.*, 2010). Pohl's team found the muon-orbited protons to be 0.84 femtometers in radius. Bezginov *et al.* (2019) announced a value equal to 0.833 fm for radius of proton in the paper "A measurement of the atomic hydrogen Lamb shift and the proton charge radius". Amazingly 2 months after report by Bezginov *et al.* (2019), newer experimental results agree with smaller measurement 0.831fm (Xiong *et al.*, 2019).

The radius of proton in measurements depends to the energy of contact and it needs a very large energy for a real contact to measure real radius of proton. Then in the future, the value of proton radius maybe reduced again in new measurements. In the next section we argue that the radius of proton is half of its Compton wavelength and it is impossible lesser and then substituting $r_p = 0.6607$ fm into Equation (172) yields to:

$$\alpha \approx 137 \quad (173)$$

This is resolution for one of the biggest puzzle of the physics, that is, so called the fine structure constant which is also verifying the size of the proton measured newly.

Richard Feynman famously thought so, saying there is a number called fine structure constant that all theoretical physicists of worth should "worry about". He called it "one of the greatest damn mysteries of physics. A magic number that comes to us with no understanding by man."

Physicist Laurence Eaves, a professor at the university of Nottingham thinks the number 137 would be the one you'd signal to the aliens to indicate that we have some measure of mastery over our planet and understand quantum mechanics.

Wolfgang Pauli joked that:

"when *i* die my first question to the devil will be: What is the meaning of the fine structure constant?" Also a joke made about the famous English physicist Paul Dirac (1902-1984), one of the founders of quantum mechanics, says that upon arrival to heaven he was allowed to ask God one question. His question was: Why 1/137?"

— Mario Livio, *The Golden Ratio: The Story of Phi, the World's Most Astonishing Number ...* Werner Heisenberg once proclaimed that all the quandaries of quantum mechanics would shrivel up when 137 was finally explained."

— Leon M. Lederman, *The God Particle: If the Universe Is the Answer, What Is the Question?*

On the other hand, from Equation (171) we have:

$$\alpha = \frac{m_p c^2}{k e^2} \times 0.21 \times 10^{-15} \quad (174)$$

Then by Compton wavelength of proton $m_p c = h/\lambda$ and $r_p = 0.6607$ fm it is deduced:

$$\alpha = \frac{\hbar c}{k e^2} \quad (175)$$

This is the same famous relation has been considered numerically for fine structure constant.

12 The Proton is a Quantum Mini Black Hole

A quantum black hole is appeared while satisfying two conditions.

a. The gravitational force prevailing to maximum centrifugal force. While m is mass of the quantum black hole and R is its radius, this is satisfied since:

$$\frac{Gm}{c^2 R} = \frac{1}{2} \quad (176)$$

This relation is equation of a quantum Schwarzschild black hole, but in reality versus a mass point potential, for a mass m with radius R , the calculation of gravitational potential in uniform sphere is showing that $G = 2G_N$. Also versus the mass point particle that stability would to appear while Compton wavelength is equal with radius which the particle is involved, while the mass is itself a volume with radius R , the least interaction and maximum attraction is occurred while,

b. Radius of particle to be half of the Compton wavelength as:

$$\lambda = 2R \quad (177)$$

Planckeons are satisfying these conditions and then the Planckeons are the example of a quantum black hole (Lutephy, 2019b). Clearly when we consider Newtonian G_N for self-gravity of proton, it doesn't result a quantum black hole whereas substituting Yukawa G_Y from Equation (169) it is deduced for proton:

$$\frac{G_Y m_p}{c^2 r_p} = \frac{1}{2} \quad (178)$$

Then the proton is a quantum black hole and then the proton existence is a strong verification for the fact that the nuclear strong force is gravity.

Now we can calculate the radius of proton exactly from second condition of a quantum black hole as:

$$r_p = \frac{\lambda}{2} = 0.6607 \text{ fm} \quad (179)$$

And this is completely matched with fine structure constant and experimental results for radius of proton.

13 The Universe zero Net Gravity

In the physics literature it has been announced that 4.6% of the total matter is observable and the total density equal to $9.9 \times 10^{-27} \text{ Kg/m}^3$.

These values are consistent to a radius for universe equal to $4.4 \times 10^{26} \text{ m}$ whereas this is inconsistent with relation $c = HR$. This paradox is emerged for considering a large portion of dark matter in the universe by scientists. But the modified gravity is responsible in the galaxy and galaxy clusters properly.

Machian mechanics certainly obeys the quantum field theory till the force having a physical meaning. Then the gravity is working in the Machian mechanics, by exchange

of the quantum particles. This is the basic and real mean of the quantum gravity. Then the gravity is working merely on the bosons and the fermions fundamentally. Then Machian dark matter should be Included to the non-observed baryons and the photons and the mesons.

The scientists are waiting to observe gravitational red shift for galaxies. But as discussed in the paper (Farley, 2010) on the data (Kowalski *et al.*, 2008; Hicken *et al.*, 2009), the observations of the red shift show the Hubble law in $H_0 = 2.23 \times 10^{18}$ as noted by Francis J. M. Farley that:

"On the largest scale, the net force accelerating or decelerating the galaxies is apparently zero."

Leading to some items supposed by Farley (2010) that:

1. "The galaxies are attracted to each other by gravity, but there is another repulsive force to cancels it.
2. Gravity falls off at large distances faster than the inverse square law.
3. Assume that the universe is isotropic on the largest scale and infinite.

But as shown in (Lyttleton and Bondi, 1959; Lutephy, 2019b), the inverse square force results contraction if attractive and expansion if repulsive. Then never we can realize the force dependent expansion from a space dependent expansion for that H in Hubble law can be additional unless by non-inverse square component of a universal net force.

Of course the universal net force is strongly zero for unlimited universe because that the observable Hubble sphere is a relative universe and its center is relative. Then gravitationally, the observable universe is center less and the gravity should be undefined unless in the gravitational bound quantum system. Indeed, the Virial theorem shows that the only combination of thermal energy and gravitational energy produces a net source for radiation in a quasi-statically contracting self-gravitating body.

But the cancellation by repulsive force seems a reality in bound quantum systems for that as discussed in (Lyttleton and Bondi, 1959) "Physical consequences of a general excess of charge", the critical charge asymmetry of the electron and proton to start expansion of universe is:

"The possibility of a general excess of charge in the universe is proposed. If such exists, even to the extent of only 1parts in 10^{18} , sufficiently powerful electric forces result to produce the observed expansion of the universe on the basis of Newtonian mechanics."

And as (Sengupta and Pal 1996), this amplitude of charge asymmetry is a probable alternative for anisotropy of cosmic microwave background radiation on Sach-Wolfe effect.

In reality cancellation point is where the gravity and repulsive force of charge asymmetry become equal, that is, the point that:

$$G = kq^2 \quad (180)$$

q is the charge of 1 kg of matter included to equal number of protons and electrons. Then we have an excess charge asymmetry for cancellation equal to:

$$e_{excess} = m_p \sqrt{G/k} = 1.4 \times 10^{-37} c \quad (181)$$

This amplitude of charge asymmetry is 2/3 that of revealed in the experiments such as Nipher electric experiments (Nipher, 1916; 1917; 1920) and also, cosmic and solar system evidences are showing it (Lutephy, 2019b) "Evidences of the charge fundamental asymmetry".

Nipher experiments on the metallic spheres show that the fundamental charge asymmetry inverts the gravity to antigravity and then repulsive force of non-neutralized baryonic matter (plasma type) is being twice as great as the pure gravity.

Littleton and Bondi suggestions are argued in the book "MOED" (Lutephy, 2016) so that for self-electric potential of the protons as the binding energy restored in the dependency of the quarks in the proton, it is appeared charge fundamental asymmetry in baryonic matter, reasoning repulsive force between individual atoms and as (Lutephy, 2019b), the plasmic repulsive force occurs for large scale (>1Mpc) for electric screening effect of the universal plasma as the same manner, the solar wind is appeared too by the repulsive electric force of the Sun on the plasma (Lutephy, 2019b) generating Parker diagrams (Parker, 1958) and also the Sun's corona by equality of the gravity and anti-gravity.

By the way substituting $c = HR$ in Machian relation yields to the total mass M so that:

$$M = \frac{c^3}{GH} = 2 \times 10^{53} kg \quad (182)$$

This relation has been derived by dimensional analysis (Valev; 2013; 2014), coinciding to the Carvalo (1995) formula obtained by totally different approach for the mass of the universe. Or equivalently in the density format, mixing $c = HR$ and Machian relation yields to:

$$\frac{4}{3} \pi G \rho = H^2 \quad (183)$$

This relation when $G=2G_N$ mimics relation in (Friedman, 1922).

14 Flattening of the Velocity Dispersion and Virial Theorem

Recent velocity measurements for several hundred stars per dSph demonstrates that dsph velocity dispersion remain approximately at with radius. For example, in (Walker *et al.*, 2007) it has been presented velocity dispersion profiles for seven dwarf satellites of the Milky Way Carina, Draco, Fornax, Leo I, Leo II, Sculptor and Sextans and all the measured dSphs exhibit approximately flat velocity dispersion profiles. Theoretical answer for this flattening of the velocity dispersion in these dwarf galaxies has been on the assumption that dSph are equilibrium systems embedded within dark matter halos however as assessed by (Walker *et al.*, 2007), the stellar velocity distributions are highly anisotropic or ongoing tidal disruption invalidates the assumption of the equilibrium (Kroupa, 1997).

One of the Milgrom paradigms is that:

"For spheroidal systems a mass-velocity-dispersion relation $\sigma^4 = a_0 GM$ is predicted under some circumstances. According to MOND, this is the fact underlying the observed Faber-Jackson relation for elliptical galaxies, which are approximately isothermal spheres (Milgrom, 1984). For instance, this relation holds approximately for all isothermal spheres having a constant velocity dispersion and constant velocity anisotropy ratio (Milgrom, 1984). in the deep MOND regime, of the form $(4/9) a_0 G_N M$, where σ is the 3-D rms velocity dispersion."

Flattening of the velocity dispersion in the galaxies is extracted in similar way with rotation curve flattening, since we have a similar relation for velocity dispersion in relation with gravity.

In reality, the velocity dispersion is the entropic motions of the bodies around a mean value which instead vector g it is correlated to a scalar form of force. For dependency of the entropy to the scalar gravitational potential, the Newtonian vector gravity is transferred to its scalar potential form as:

$$\varphi = \sigma^2 \quad (184)$$

And for mean central scalar gravitational potential by Equation (27) applied in Equation (184) we deduce:

$$\varphi_{tot} = M_{tot} \bar{\sigma}^2 \quad (185)$$

And this is Virial theorem.

The total potential energy in extended form is written as follows:

$$\bar{\phi}_{tot} = \int \rho(g \bullet r) dV \quad (186)$$

Milgrom (1984; 1994) has used complicate calculations to extract velocity dispersion of the isolated isothermal spheres. But we use here a simple way. Substituting Milgrom gravity formula into the Equation (186) yields to:

$$\bar{\phi}_{tot} = 4\pi \int \rho \sqrt{a_0 g} r^3 dr \quad (187)$$

For isothermal spheres we have $\rho = kr^{-2}$ and substituting this density profile into Equation (187) we result:

$$\bar{\phi}_{tot} = 4\pi k \sqrt{a_0 G} \int \sqrt{M_r} dr \quad (188)$$

For isothermal sphere we have $M_{cr} = 4\pi kr$ and substituting this equation into Equation (188) results:

$$\bar{\phi}_{tot} = \sqrt{a_0 G} \times \frac{2}{3} M^{3/2} \quad (189)$$

Substituting Equation (189) into the Equation (185) results:

$$\frac{2}{3} \sqrt{a_0 G_N M} = \sigma \quad (190)$$

Deviation from isothermal sphere leads to the general inequality $\frac{4}{9} \leq \frac{\delta^4}{a_0 G_N M} \leq 1$ which has been reported in (Milgrom, 1984; Sanders, 2010). Of course Milgrom's MOND is not match perfectly with galaxies velocity dispersions for that Milgrom's gravity is equivalent with Faber-Jackson relation (Faber and Jackson, 1976) and then since a galaxy does deviate from Faber-Jackson relation, the Milgrom's gravity too does deviate.

Conclusion

We have modified the Newtonian mutual gravity by Mach's inertia principle and generalized to quantum bound systems and finite-range limitation of force carriers.

The paper matches with experimental reports about the galaxies rotation curve and velocity dispersions, verifying Machian version of the Newton's gravity here. In reality not only in pure arguments we refer directly to the scientific community suppose the results are confirmed closest to the scientific reports. The Machian MOND reduces to Milgrom MOND for standard bound systems, that is, the systems following the so called Sersic profiles, without any hypothetical acceleration considered by Mordehai Milgrom.

Too this paper is as a dictionary for the knowledge of Machian universe and its method in the physics and philosophy. The paper is inasmuch as logical which we don't await unless a general acceptance.

We have argued the origin of the size of the proton's mass, origin of the nuclear saturation properties, origin of the Yukawa strong force as the strong gravity, origin and argument of the big question, that is, the fine structure constant, true estimation of the universe total matter, argument for exact cancellation of the universal inverse square net gravity by repulsive force of fundamental charge asymmetry of baryonic matter, generalization of Fish's law, generalization of Tully-Fisher and Faber-Jackson relations, very beautiful argument for Fish's and Freeman laws as the constancy of the central extrapolated surface density of the galaxies, generalization of Machian relation in quantum bound systems, arguing the Milgrom phenomenological paradigms, very complex argument of Kormendy relation.

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Ethics

No part of manuscript reporting original work is being considered for publication in whole or in part elsewhere.

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