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Plasma Beams Free Vibration Ivestigation using the Boubaker Polynomials Expansion Scheme

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Abstract: Problem statement: In the present study, plasma beams free vibration frequencies alteration has been investigated through an original protocol. **Approach:** The Boubaker Polynomials Expansion scheme BPES has been performed for deducing and ranging the dimensionless frequencies of the beam vibrations. **Results:** Natural frequencies of the plasma beam have been estimated for given parameters values. **Conclusion:** Yielded results have been compared and discussed. It was found that a good agreement as the values determined with experimental results determined by previous researchers.

Key words: Plasma beams, free vibration, natural frequency, mechanical vibrations, Metastability Exchange Optical Pumping (MEOP), avalanche ionization, generally laminated plasma

INTRODUCTION

In the two last decade, plasma beams have been given interesting electric and fire properties (Dohnalik *et al*., 2011; Vo *et al*., 2010; Ghanouchi *et al*., 2008; Chen 2003; Qindeel *et al*., 2007; Khare *et al*., 2004; Della and Shu, 2005; Kisa *et al*., 1998; Meirovitch, 1986). Sollier *et al*. (2011) studied plasma beems vibration patterns in terms of peak irradiance, pulse width and duration for energy pulses transmitted through the breakdown plasmas generated in fluids. Recorded data for 25 ns-1064 nm beam pulses outlined the roles of avalanche ionization and multiphoton ionization. Vo *et al*. (2010) studied the Metastability Exchange Optical Pumping (MEOP) performed at elevated 3He gas pressures using an annular beam and elaborated an accurate vibration model profiling. Many analytical methods of analysis have been used to study the vibration of plates, shells and beams (Khare *et al*., 2004; Della and Shu, 2005; Kisa *et al*., 1998; Meirovitch, 1986; Sollier *et al*., 2011; Osman and Hora, 2004; Osman *et al*., 2004; 2005).

In this study, a model on the vibration analysis of plasma beam has been developed and studied using a confirmed and tested scheme under some presumptions.

MATERIALS AND METHODS

According to the model presented in Fig. 1, The relationship between normal stress and bending moment, according to Bernoulli-Euler hypotheses, is given by Eq. 1:

$$M = \frac{2h}{3\rho}\sum_{k=1}^{N} E_x (z_k^3 - z_{k-1}^3) \quad (1)$$

Where:
ρ = Beam curvature.
H = Beam radius.
N = The number of layers
Z_k = k^{th} layer outer face distance to neutral plane
E_x = Elasticity modulus

Consequently, the relationship between the bending moment and the curvature can be written as Eq. 2 follows:

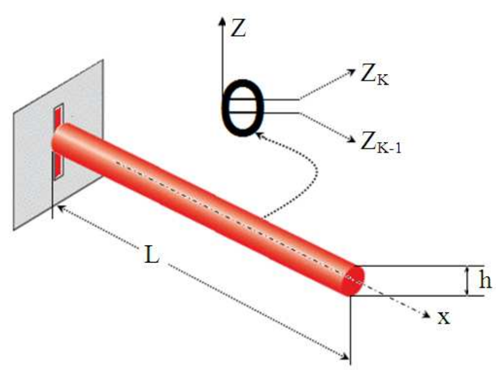

Fig. 1: Geometrical model of the studied beam

$$\left\{ M = \frac{E_{ef} I_{yy}}{\rho} = E_{ef} I_{yy}\frac{d^2W}{dx^2},\ E_{ef} = \frac{8}{h^3}\sum_{k=1}^{N/2} (E_x)_k (z_k^3 - z_{k-1}^3) \right. \quad (2)$$

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Where:
E_{ef} = Effective elasticity modulus.
I_{yy} = Beam cross-sectional inertia moment.
W = Lateral deflection *x*-dependent mode profile

Plasma beam flexural motion is described by the Eq. 3:

$$E_{ef} I_{yy} \frac{\partial^4 w(x,t)}{\partial x^4} + \rho \frac{\partial^2 w(x,t)}{\partial t^2} = 0 \quad (3)$$

With a trivial solution Eq. 4:

$$w(x,t) = e^{j\varpi_n t} W(x) \quad (4)$$

where, ϖ_n is the frequency.

By introducing the expression (4) in Eq. 3, the following time-independent non-variable characteristic value problem is deduced Eq. 5:

$$\frac{d^4 W(x)}{dx^4} - f_n^2 W(x) = 0 \quad (5)$$

where, f_n is the dimensionless frequency of the beam vibrations given by Eq. 6:

$$f_n^2 = \frac{\varpi^2 \rho}{E_{ef} I_{yy}} \quad (6)$$

Along with the boundary conditions, as per Eq. 7 Shu (2000):

$$\begin{cases} W(x)\big|_{x=0} = 0 \\ \dfrac{dW(x)}{dx}\bigg|_{x=0} = 0 \\ \dfrac{d^2 W(x)}{dx^2}\bigg|_{x=L} = 0 \end{cases} \quad (7)$$

The solution of the system referring to Eq. 4 and 6 is obtained using the Boubaker Polynomials Expansion Scheme BPES (Awojoyogbe and Boubaker, 2009; Labiadh and Boubaker, 2007; Slama *et al*., 2008; 2009; Hossein *et al*., 2009; Fridjine and Amlouk, 2009; Belhadj *et al*., 2009a; 2009b), through establishing the expression Eq. 8:

$$W(x) = \frac{1}{2N_0} \sum_{k-1}^{N0} \lambda_k \times B_{4k}(x \times \frac{r_k}{L}) \quad (8)$$

where, B_{k4} are the 4k-order Boubaker polynomials, $x\in]0,L[$ is the normalized distance, r_k are B_{k4} minimal positive roots, N_0 is a prefixed integer and $\lambda_k\big|_{k=1,..N_0}$ are unknown pondering real coefficients.

Consequently, it comes for Eq. 5 that Eq. 9:

$$\frac{1}{2N_0}\left(\frac{r_k}{L}\right)^4 \sum_{k=1}^{N_0} \lambda_k \times \frac{dB_{4k}}{dx^4}(x \times \frac{r_k}{L}) - \xi^2 \frac{1}{2N_0}\left(\frac{r_k}{L}\right)^4 \sum_{k=1}^{N_0} \lambda_k \times B_{4k}(x \times \frac{r_k}{L}) = 0 \quad (9)$$

The related boundary conditions expressed through Eq. 6. The Boubaker Polynomials Expansion Scheme (BPES) protocol ensures their validity regardless main equation features. In fact, thanks to Boubaker polynomials first derivatives properties are Eq. 10 and 11:

$$\begin{cases} \sum_{q=1}^{N} B_{4q}(x)\big|_{x=0} = -2N \neq 0 \\ \sum_{q=1}^{N} B_{4q}(x)\big|_{x=r_q} = 0 \end{cases} \quad (10)$$

And:

$$\begin{cases} \sum_{q=1}^{N} \dfrac{dB_{4q}(x)}{dx}\bigg|_{x=0} = 0 \\ \sum_{q=1}^{N} \dfrac{dB_{4q}(x)}{dx}\bigg| x = r_q = \sum_{q=1}^{N} H_q \end{cases}$$
$$\text{with}: H_n = B_{4n}(r_n)\left[\frac{4r_n\left[2 - r_n^2\right] \times \sum_{q=1}^{n} B_{4q}^2(r_n)}{B_{4(n+1)}(r_n)}\right] \quad (11)$$

Boundary conditions are inherently verified Eq. 12:

$$\begin{cases} \dfrac{dW(x)}{dx}\bigg|_{x=0} = \dfrac{1}{2N_0}\sum_{k=1}^{N_0} \lambda_k \times \dfrac{dB_{4k}(x)}{dx}\bigg|_{x=0} = 0 \\ \dfrac{dW(x)}{dx}\bigg|_{x=L} = \dfrac{1}{2N_0}\sum_{k=1}^{N_0} \lambda_k \times \dfrac{db_{4k}(x)}{dx}\bigg|_{x=rk} = 0 \\ \sum_{k=1}^{N_0} \lambda_k \times H_n = 0 \end{cases} \quad (12)$$

The BPES solution is obtained by integrating, for a given value of N_0, the whole expression given by Eq. 8

Table 1: Beam frequencies estimated using the PBES

Mode No.	Frequency	Error (%)
1	49.85	2.3
2	363.53	2.8
3	998.93	2.4
4	1090.45	1.9
5	1210.55	2.2
6	1305.92	2.1
7	1403.15	2.0
8	1500.49	2.1

along the interval [0, L], determining the set of coefficients where $\tilde{\lambda}_k\big|_{k=1,..N_0}$ that minimizes the absolute difference Eq. 13:

$$\begin{cases} \Phi = \dfrac{1}{2N_0}\left|\left(\sum_{k=1}^{N_0}\tilde{\lambda}_k \times \alpha_k\right) - \zeta\left(\sum_{k=1}^{N_0}\tilde{\lambda}_k \times \beta_k\right)\right| \\ \text{with:} \\ \alpha_k = \left(\dfrac{r_k}{L}\right)^4 \displaystyle\int_0^L \dfrac{dB_{4k}}{dx^4}(x \times \dfrac{r_k}{L})dx \\ \beta_k = \displaystyle\int_0^L B_{4k}(x \times \dfrac{r_k}{L})dx \end{cases} \quad (13)$$

And finally deducing, for increasing values of N_0, the corresponding frequency using Eq. 6.

RESULTS

Plasma beam frequencies estimated using the boubaker polynomials expansion scheme PBES are gathered in Table 1.

DISCUSSION

The natural frequency changes as a direct result of the change in the stacking sequence causes resonance if the changed frequency becomes closer to the working frequency. This feature is in good agreements with the results published by Aly *et al*. (2010); Alsultanny (2006); Mardani (2008) and Hashemi *et al*. (2007).

With relation to the deviations of the numerical results in relation to those recorded in the related literature, some possible measurement errors can be pointed out such as measurement noise or non-uniformity in the beam properties (voids, variations in radius, non uniform outer shape ..). Such factors are not taken into account during the numerical analysis, since the model considers a strictly homogeneous beam, what rarely occurs in practice.

CONCLUSION

This study presents a protocol for the calculation of natural frequency of the plasma beam. Calculations performed by means of Boubaker Polynomials Expansion Scheme (PBES) resulted in coherent similar results. Changes in the stacking sequence as well as damping effects, which can have a large influence on the path of the beam, are the subject of coming investigations.

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