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GRAPH OF FINITE SEQUENCE OF FUZZY TOPOGRAPHIC TOPOLOGICAL MAPPING OF ORDER TWO

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ABSTRACT

Fuzzy Topographic Topological Mapping (FTTM) was built to solve the neuromagnetic inverse problem to determine the location of epileptic foci in epilepsy disorder patient. The model which consists of topological and fuzzy structures is composed into three mathematical algorithms. FTTM consists of four topological spaces and connected by three homeomorphisms. FTTM version 1 is also homeomorphic to FTTM version 2. This homeomorphism generates another 14 elements of FTTM. In this study we proved that, if there exist n elements of FTTM, the new elements of order 2 will produce a graph of degree $24n^2$-16n-8. In this study, the statement is proven by viewing FTTMs as sequence and using its graphical features. In the process, several definitions and theorems were developed.

Keywords: Fuzzy Topographic Topological Mapping, Sequence of $FTTM_n$, Element of Order Two, M_1

1. INTRODUCTION

FTTM version 1 consists of three algorithms, which link between four components. The four components are magnetic contour plane (M_1), base magnetic plane (B_1), fuzzy magnetic field (F_1) and topographic magnetic field (T_1), as shown in **Fig. 1**.

FTTM version 1 was developed to present a 3-D view of unbounded signal current source (Fauziah, 2002; Liau, 2001).

Besides that, FTTM version 2 can processed image data of magnetic field.

FTTM version 1 as well as FTTM version 2 are specially designed to have equivalent topological structures between its components (Yun, 2006). In other words, there are homeomorphisms between each element of FTTM version 1 and FTTM version 2 (Yun, 2006) (**Fig. 2**).

Yun (2006) first noticed that if there were two elements of FTTM that are homeomorphic to each other component wise, it would generate more homeomorphisms. The numbers of generating new elements of FTTM are:

$$\left[\binom{2}{1}\times\binom{2}{1}\times\binom{2}{1}\times\binom{2}{1}\right]-2=14\,\text{elements}$$

Consequently, Yun (2006) proposed a conjecture such that, if there exist n elements of FTTM, then the number of new elements are n^4-n.

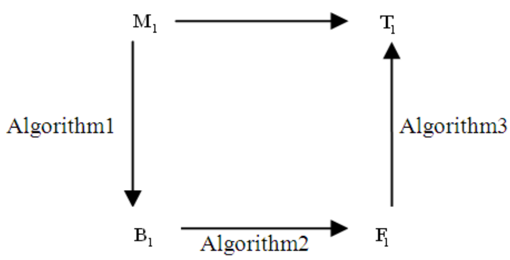

Fig. 1. FTTM Version 1

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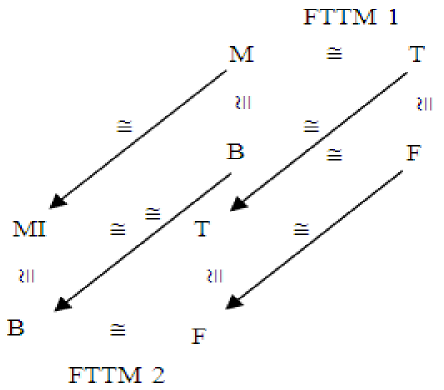

Fig. 2. Homeomorphisim Between FTTM 1 and FTTM 2

The above conjecture finally proved by (Ahmed *et al*., 2010) as stated below.

Theorem 1

If there exist n elements of FTTM, the numbers of new elements are n^4-n.

2. MATERIALS AND METHODS

Definition 1: (Ahmed *et al*., 2010)

Let $FTTM_i = (M_i, B_i, F_i, T_i)$ such that M_i, B_i, F_i and T_i are topological spaces with M_i @ B_i @ F_i @ T_i. Sequence of n $FTTM_i$ of FTTM is $FTTM_1, FTTM_2, FTTM_3, ..., FTTM_n$, such that M_i @ M_{i+1}, B_i @ B_{i+1}, F_i @ F_{i+1}, T_i @ T_{i+1} (**Fig. 3**).

In this study we present a new notion of sequence of FTTM namely "Order of the new elements" and this study will deal with graph of the new elements of order two.

In the following definition cube order of FTTM is presented.

Definition 2

The cube that produced from a combination of $FTTM_i$ in $FTTM_n$ is said to be a cube of order i and i = 2, 3, 4.

It is impossible to develop cubes from the combination of five or more terms FTTM.

Notation

- $C_{i,j}$ $FTTM_i$ presents the cubes of order two that can be produced from the combination of $FTTM_i$ in $FTTM_n$ in $FTTM_n$, where $1 \leq i < j \leq n$:
- I = {1,2,3,4,…,n-1}
- J = {2,3,4,…,n}

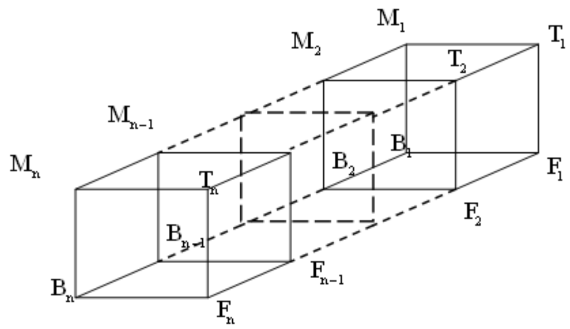

Fig. 3. Sequence of FTTM

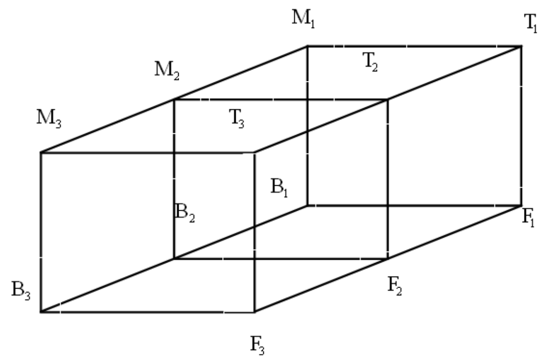

Fig. 4. $FTTM_3$

- $\left|C_{i,j}FTTM_n\right|_{i\in I, j\in J, i<j\leq n}$ presents, the number of cubes of order two that can be produced from the combination of $FTTM_i$ and $FTTM_j$ in $FTTM_n$, such that:

$$i \in I,\ j \in J,\ \forall\ i < j \leq n$$

Example 1

In $FTTM_3$ From **Fig. 4** we can find that:

$$C_{i,j}FTTM_3 = C_{1,2}\,FTTM_3,\ C_{2,3}\,FTTM_3,\ C_{1,3}FTTM_3.$$

$$\left|C_{i,j}FTTM_3\right|_{i\in I, j\in J, i<j\leq 3} = 3$$

Now we have the following lemma.

Lemma 1

$$\left|C_{i,j}FTTM_n\right|_{i\in I, j\in J, i<j\leq n} = 1+2+3+\ldots+(n-2)+(n-1) = \frac{n(n-1)}{2}$$

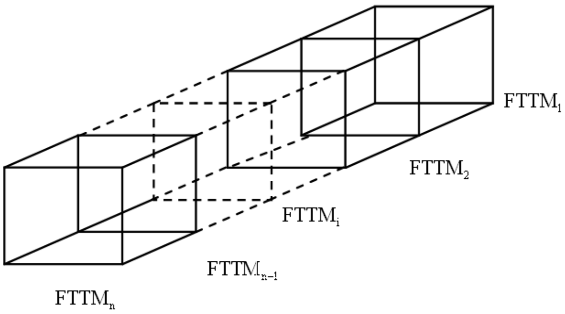

Fig. 5. Sequence of $FTTM_n$

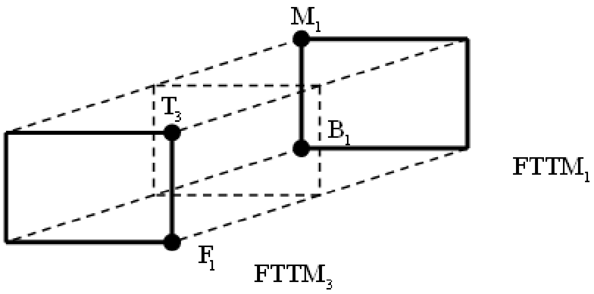

Fig. 6. (M_1, B_1, F_3, T_3)

Proof

From **Fig. 5**:

$$
\begin{aligned}
S_1 &= \left|C_{i,i+1}FTTM_n\right|_{i\in I,\, j\in J, i<j\leq n} = n-1 \\
S_2 &= \left|C_{i,i+2}FTTM_n\right|_{i\in I,\, j\in J, i<j\leq n} = n-2 \\
&\vdots \qquad\qquad \vdots \\
S_{n-2} &= \left|C_{i,n-2}FTTM_n\right|_{i\in I,\, j\in J, i<j\leq n} = 3 \\
S_{n-1} &= \left|C_{i,n-1}FTTM_n\right|_{i\in I,\, j\in J, i<j\leq n} = 2 \\
S_n &= \left|C_{i,n}FTTM_n\right|_{i\in I,\, j\in J, i<j\leq n} = 1
\end{aligned}
$$

Now let:

$$S=\sum_{i=1}^{n} S_i$$

Therefore:

$$
\begin{aligned}
2S = 1+2+3+\cdots \\
+(n-2)+(n-1) \\
+(n-1)+(n-2) \\
+\cdots+2+1
\end{aligned}
$$

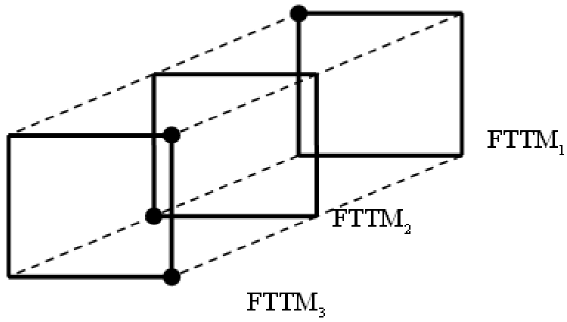

Fig. 7. (M_1, B_1, F_3, T_3)

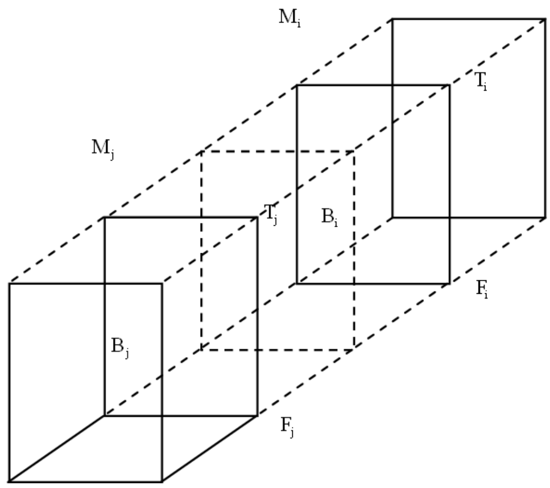

Fig. 8. $FTTM_i$ and $FTTM_j$

$$
\begin{aligned}
&= n(n-1) \\
S &= \frac{n(n-1)}{2}
\end{aligned}
$$

Here we introduce a new definition.

Definition 3

The new element is said to be an element of order I if its components appear in exactly i versions of $FTTM_n$. i = 2, 3, 4.

Example 2

(M_1, B_1, F_3, T_3) is an element of order two, since its components appear in $FTTM_1$ and $FTTM_3$ (**Fig. 6**).

By replacing B_1 with B_2, then (M_1, B_1, F_3, T_3) is an element of order three, since its components appear in $FTTM_1$, $FTTM_2$ and $FTTM_3$, as presented in **Fig. 7**.

The following lemma is immediate.

Lemma 2

Any $C_{i,j}FTTM_n$ will generate 14 new elements of order two.

Proof

Without loss of generality, consider $FTTM_1$ and $FTTM_j$ where ($i\in I$, $j\in J$, $i<j\leq n$) as in **Fig. 8**.

The new elements can be generated by simple construction, as given below:

$$(M_i,B_i,F_i,T_j),(M_i,B_i,F_j,T_i)$$
$$(M_i,B_j,F_i,T_i),(M_j,B_i,F_i,T_i)$$
$$(M_i,B_i,F_j,T_j),(M_i,B_j,F_i,T_j)$$
$$(M_j,B_i,F_i,T_j),(M_i,B_j,F_j,T_i)$$
$$(M_j,B_i,F_j,T_i),(M_j,B_j,F_i,T_i)$$
$$(M_i,B_j,F_j,T_j),(M_j,B_j,F_i,T_j)$$
$$(M_j,B_j,F_j,T_i),(M_j,B_i,F_j,T_j)$$

All the above elements are of order two.

Notation

Some new notations are introduced as follows:

- $\alpha_a FTTM_n$ presents the set of all elements of order α that can be generated by $FTTM_n$
- $\beta_a FTTM_n$ presents the set of all elements of order other than α, that can be generated by $FTTM_n$

Consequently, the following theorem is presented as below.

Theorem 2

$$|\alpha_2 FTTM_n| = 7(n^2-n) \text{ and}$$
$$|\beta_2 FTTM_n| = n^4-7n^2+6n$$

Proof

From Definition 2, Lemma 1 and Lemma 2:

$$|\alpha_2 FTTM_n| = 14\times |C_{i,j}FTTM_n|_{i\in I, j\in J, i<j\leq n}$$
$$= 14\times\frac{n(n-1)}{2}$$
$$= 7n(n-1)$$
$$= 7(n^2-n)$$

Table 1. Elements of Order Two

N	$\lvert\alpha_2 FTTM_n\rvert$	$\lvert\beta_2 FTTM_n\rvert$	n^4-n
1	0	0	0
2	14	0	14
3	42	36	87
4	84	168	252
5	140	480	620
6	210	1080	1290
7	294	2100	2394
8	392	3696	4088
9	504	6048	6552
10	630	9360	999

Now by subtracting the number of the new elements of order two from the total number of new elements then the number of new elements of order other than two (three and four) can be deduced as follows:

$$|\beta_2 FTTM_n| = n^4-n-7(n^2-n)$$
$$= n^4-n-7n^2+7n$$
$$= n^4-7n^2+6n$$

2.1. Computational Results

In the previous discussion we show that the number of the new elements of order two equal to $7(n^2$-n) and the number of the new elements of order other than two equal to n^4-$7n^2$+6n.

The number of the new elements of order two and of order other than two, for n = 1,2,3,…,10 is given in the following **Table 1**.

Lemma 3

Any element of order two will produce a graph of degree 4 or zero.

Proof

Since the element is of order two, its graph will appear in exactly two versions of FTTM in $FTTM_n$. From Lemma 2, $FTTM_i$ and $FTTM_j$ will generate 14 new elements of order two and of 12 of them will produce a graph of degree 4, namely:

$$(M_i,B_i,F_i,T_j),(M_i,B_i,F_j,T_i)$$
$$(M_i,B_j,F_i,T_i),(M_j,B_i,F_i,T_i)$$
$$(M_i,B_i,F_j,T_j),(M_j,B_i,F_i,T_j)$$
$$(M_i,B_j,F_j,T_i),(M_j,B_i,F_j,T_i)$$
$$(M_i,B_j,F_j,T_j),(M_j,B_j,F_i,T_j)$$
$$(M_j,B_j,F_j,T_i),(M_j,B_i,F_j,T_j)$$

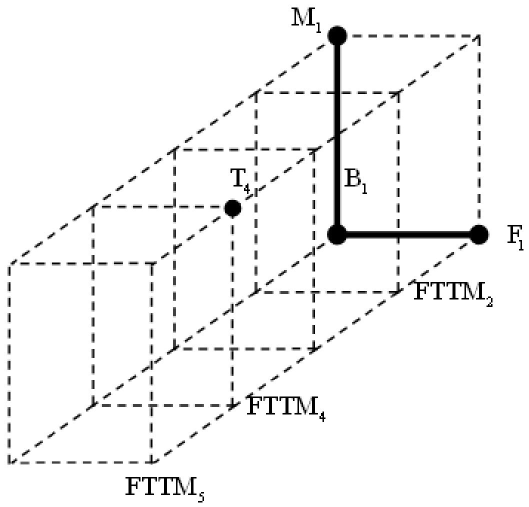

Fig. 9. (M_1, B_1, F_1, T_4)

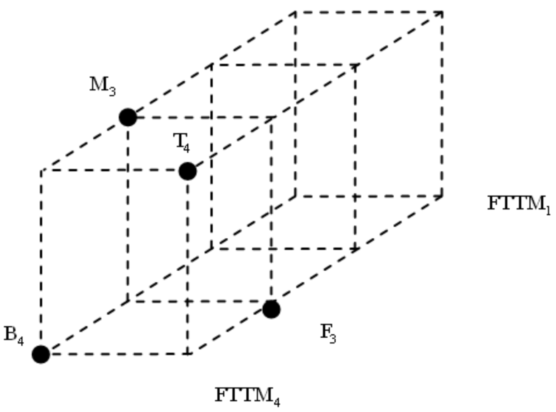

Fig. 10. (M_1, B_1, F_1, T_4)

And 2 elements will produce a graph of degree zero, (M_i, B_j, F_i, T_j) and (M_j, B_i, F_j, T_i) since all its vertices are isolated (No homeomorphism between its components).

Example 3

Any element of order two of the form (M_i, B_j, F_i, T_j) will produce a graph of degree four, $\forall\ i < j \le n$.

To see this let $i = 2\ \ , j = 4\ , n = 5$, (**Fig. 9**).

The degree of the graph:

$$
\begin{aligned}
&= \deg(v_1) + \deg(v_2) + \deg(v_3) + \deg(v_4) \\
&= \deg(M_1) + \deg(B_1) + \deg(F_1) + \deg(T_4) \\
&= 1 + 2 + 1 + 0 \\
&= 4
\end{aligned}
$$

Example 4

Any element of order two of the form (M_i, B_j, F_i, T_j) will produce a graph of degree zero, $\forall\ i < j \le n$.

To see this let $i = 3\ \ , j = 4\ , n = 4$, (**Fig. 10**).

The degree of the graph:

$$
\begin{aligned}
&= \deg(v_1) + \deg(v_2) + \deg(v_3) + \deg(v_4) \\
&= \deg(M_3) + \deg(B_4) + \deg(F_3) + \deg(T_4) \\
&= 0 + 0 + 0 + 0 \\
&= 0
\end{aligned}
$$

3. RESULTS AND DISCUSSION

Finally, the new result follows.

Theorem 3

The new elements of order two will construct a graph of degree $24n^2$-16n-8.

Proof

From Theorem 2, Lemma 1, Lemma 2 and Lemma 3 Equation 1:

$$
\begin{aligned}
&\mathrm{Deg}(\alpha_2 FTTM_n) = \frac{n(n-1)}{2} \times (12) \times (4) + H \\
&= 24(n^2 - n) + H
\end{aligned}
\tag{1}
$$

Such that H is the number of added edges to the graph, since $FTTM_i$ and $FTTM_{i+1}$ are homeomorphic component wise, therefore Equation 2:

$$
\begin{aligned}
&H = 2(4) + 8(n-2) \\
&= 8 + 8n - 16 \\
&= 8n - 8
\end{aligned}
\tag{2}
$$

Substitute (2) into (1):

$$
\begin{aligned}
&\mathrm{Deg}(\alpha_2 FTTM_n) = 24(n^2 - n) + 8n - 8 \\
&= 24n^2 - 16n - 8
\end{aligned}
$$

4. CONCLUSION

In this study, it is proven that if there exist N elements of FTTM of order two, then it will produce a graph of degree $24n^2$ -16n-8.

5. REFERENCES

Ahmed, T., S.S. Jamian and J. Talib, 2010. Generalized finite sequence of fuzzy topographic topological mapping. Math. Stat., 6: 151-156. DOI: 10.3844/jmssp.2010.151.156

Fauziah, Z., 2002. Algorithma Penyelesan Masala Songsang Arus Tunggal Tak Terbatas MEG. University Teknologi Malaysia, Tesis Sarjana.

Liau, L.Y., 2001. Homeomorfisma S2 Antara E2 Melalui Struktur Permukaan Riemann serta Deduksi Teknik pembuktiannya bagi Homeomorfisma pemetaan Topologi Topografi Kabur (FTTM). University Teknologi Malaysia, Tesis Sarjana.

Yun, L.L., 2006. Group-like algebraic structures of fuzzy topographic topological mapping for solving neuromagnetic inverse problem. Ph.D. Thesis, University Technology Malaysia.