COST-BENEFIT ANALYSIS OF A SINGLE UNIT SYSTEM WITH SCHEDULED MAINTENANCE AND VARIATION IN DEMAND

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ABSTRACT

The present paper analyses the reliability and cost-benefit for a single unit system with scheduled maintenance and variation in demand. As Change in demand affects the production of system also, hence sometimes, the system needs to be shut down when the number of produces are in excess as compared to those demanded. Revenue in case of both types of up states i.e., when demand is greater than or equal to production and when demand is less than production have also been taken under consideration while carrying out the cost-benefit analysis. The loss incurred to the system when it is kept shut down due to less demand has also been taken into account. Optimized reliability indices of the system effectiveness are estimated numerically using semi-Markov processes and regenerative point technique. Expression for the expected profit is obtained after obtaining various measures of system effectiveness. We can conclude that cut off points for various rates/probabilities/revenue per unit up time/costs can be obtained which help in deciding the upper/lower acceptable values of rates/costs so that the system is profitable.

Keywords: Single Unit System, Scheduled Maintenance, Variation in Demand, Regenerative Point Technique, MTSF, Cost-Benefit Analysis

1. INTRODUCTION

Literature of Reliability contains lot of studies on the reliability and cost-benefit analysis of various systems. These studies are contributed by various researchers including Rizwan *et al.* (2010); Manocha *et al.* (2011) and Kumar and Kumar (2012) where in the concepts of operating and rest periods, hot standby Programmable Logic Controller, random inspection, instructions, ash water pump systems with and without failed states, hardware based software interaction failures and different types of recovery have been taken up. These studies have considered the demand as fixed. However, there exist many practical situations where the demand of the units produced is not fixed. Such a situation may be seen in General Cable Energy System and hence there is need of studying reliability and availability analysis of a system

with varying demand and hence the present paper. The present paper investigates the reliability and cost-benefit analysis of a single unit system with scheduled maintenance and variation in demand. As variation in demand affects the production and hence the system is required to be put to down state when the units produced are already in excess. The system in the down state is made operative as soon as the produced units are less in number than those demanded.

2. MATERIALS AND METHODS

In this study, the probabilistic analysis of the system is analyzed by making use of semi-Markov processes and regenerative point technique and have obtained various measures of system effectiveness such as Mean Time to System Failure, The Steady State Availability

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when demand is not less than production, The Steady State Availability when demand is less than production, Busy period of the repairman for repair at t = 0, Expected number of visits by the repairman at t = 0, Expected Number of scheduled maintenances at t = 0, Expected down time at t = 0, Profit incurred to the system.

2.1. Notations

λ = Failure rate of the operative unit

λ_1 = Rate of decrease of demand so as to become less than production

λ_2 = Rate of increase of demand so as to become greater than or equal to production

λ_3 = Rate of going from upstate to downstate (reason behind this is that the demand is less than production and production goes on increasing and as a result we have lot of produces in the stock. This production needs to be stopped

λ_4 = Rate of change of state from down to up when there is no produce with the system and demand is there

β_1 = Rate of requirement of doing scheduled maintenance

β_2 = Rate of doing scheduled maintenance

p_1 = Probability that during the repair time the demand is greater than or equal to production

p_2 = Probability that during the repair time the demand is less than production

Fr = Failed unit under repair

A_0^d = Steady state availability of the system when demand is greater than or equal to production

A_0^p = Steady state availability of the system when demand is less than production

$B_0^0(t)$ = Busy period of the repairman for repair at $t = 0$

$V_0^0(t)$ = Expected number of visits by the repairman at $t = 0$

$SM_0^0(t)$ = Expected number of schedule maintenance at $t = 0$

$DT_0^0(t)$ = Expected down time at $t = 0$

$M_i(t)$ = Probability that system up initially in regenerative state i is up at time t without passing through any other regenerative state

m_{ij} = Contribution to mean sojourn time in regenerative state i before transiting to regenerative state j without visiting to any other state $\mu_i(t)$ Mean sojourn time in regenerative state before transiting to any other state

$*$ = Symbol for Laplace transforms

$**$ = Symbol for Laplace Stieltjes transforms

$©$ = Symbol for Laplace convolution

$\circledS$ = Symbol for Stieltjes convolution

$q_{ij}(t), Q_{ij}(t)$ = p.d.f. and c.d.f of first passage time from a regenerative state i to a regenerative state j or to a failed state j without visiting any other regenerative state in (0, t)

$g(t), G(t)$ = p.d.f. and c.d.f. of repair time for the unit

2.2. Symbols for the States of System

O_{SM} = Operative unit under online scheduled maintenance

$Op_{d \geq p}$ = Operative unit when demand is not less than production

$Op_{d < p}$ = Operative unit when demand is less than production

D = Notation for down unit

Fr = Failed unit under repair

3. RESULTS

For the particular case, the rate of repair (α) is assumed to be exponentially distributed. Let us take $g(t) = \alpha e^{-\alpha t}$. Various estimated values on the basis of collected data are:

$$\lambda = 0.005, \lambda_1 = 1, \lambda_2 = 1, \lambda_3 = 0.1,$$
$$\lambda_4 = 5, \alpha = 2, \beta_1 = 0.2\ \beta_2 = 4$$

By taking the values of $p_1 = 0.660$, $p_2 = 0.340$, $C_0 = 1200$, $C_1 = 500$, $C_2 = 100$, $C_3 = 200$, $C_4 = 200$, $C_5 = 200$, graphical study is carried out.

3.1. Transition Probabilities and Mean Sojourn Times

The transition diagram showing the various states of the system is shown as in **Fig. 1**. The epochs of entry into states S_0, S_1, S_3 and S_5 are regeneration points and thus are regenerative states. States S_2 and S_4 are failed states. The transition probabilities are:

$$q_{01}(t) = \lambda_1 e^{-(\lambda + \lambda_1 + \beta_1)t},\ q_{02}(t) = \lambda e^{-(\lambda + \lambda_1 + \beta_1)t},$$
$$q_{03}(t) = \beta_1 e^{-(\lambda + \lambda_1 + \beta_1)t},\ q_{10}(t) = \lambda_2 e^{-(\lambda_2 + \lambda_3 + \lambda)t},$$
$$q_{14}(t) = \lambda e^{-(\lambda_2 + \lambda_3 + \lambda)t},\ q_{15}(t) = \lambda_3 e^{-(\lambda_2 + \lambda_3 + \lambda)t},$$
$$q_{20}(t) = g(t),\ q_{30}(t) = \beta_2 e^{-\beta_2 t},\ q_{40}(t) = p_1 g(t),$$
$$q_{41}(t) = p_2 g(t),\ q_{50}(t) = \lambda_4 e^{-\lambda_4 t}$$

The non-zero elements p_{ij} are obtained as:
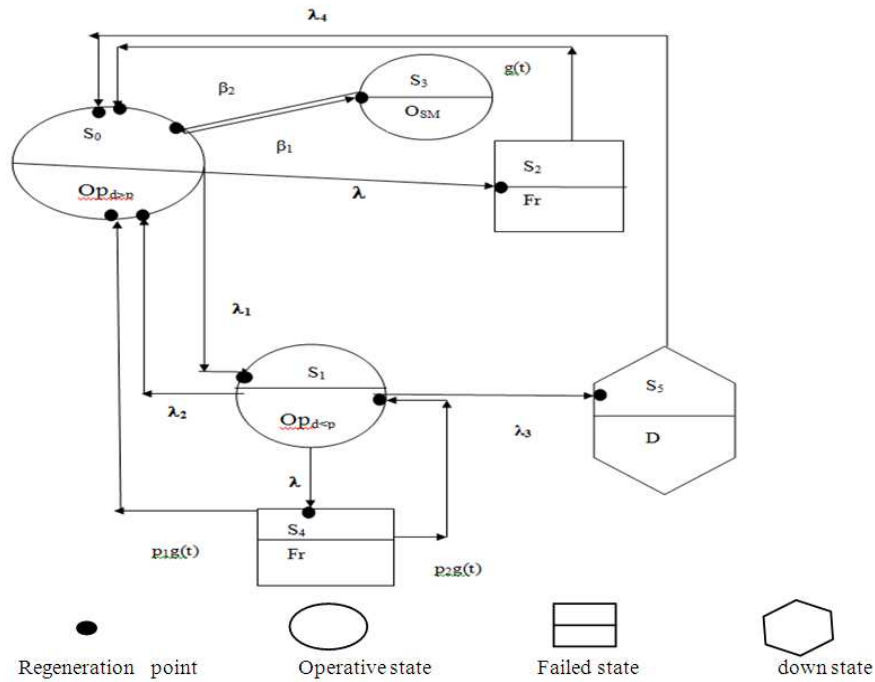
$$p_{ij} = \lim_{s \to 0} q_{ij}^*(s)$$

Fig. 1. State transition diagram

The mean sojourn time (μ_i) in state i, are:

$\mu_0 = 1 / (\lambda + \lambda_1 + \beta_1)$; $\mu_1 = 1 / (\lambda + \lambda_2 + \lambda_3)$;
$\mu_2 = - g^{*'}(0)$; $\mu_3 = 1/ \beta_2$; $\mu_4 = - g^{*'}(0) = \mu_2$; $\mu_5 = 1/ \lambda_4$

The unconditional mean time taken by the system to transit for any state j when it is counted from epoch of entrance into state i is mathematically stated as:

$$m_{ij} = \int_0^\infty t \, q_{ij}(t) \, dt = - q_{ij}{*}'(0)$$

Thus, $m_{01} + m_{02} + m_{03} = \mu_0$; $m_{10} + m_{14} + m_{15} = \mu_1$; $m_{20} = \mu_2$; $m_{30} = \mu_3$; $m_{40} + m_{41} = \mu_4$; $m_{50} = \mu_5$.

4. DISCUSSION

It can be concluded from **Fig. 2 and 3** that availability decreases with increase in the values of failure rate and increase s with increase in the values of repair rate. **Figure 4** shows the behavior of profit with respect to revenue per unit up time (C_0) for different values of cost per visit of the repairman (C_3). It can be concluded from the graph that the profit increases with the increase in values of C_0 and has lower values for higher values of C_3. It is also observed from the graph that for $C_3 = 100$, the profit is positive or zero or negative

according as $C_0 >$ or $= < 500$. So, the system is profitable only if $C_0 > 500$. For $C_3 = 6100$, the profit is positive or zero or negative according as $C_0 >$ or $= < 570$. So, the system is profitable only if $C_0 > 570$. For $C_3 = 12100$, the profit is positive or zero or negative according as $C_0 >$ or $= < 625$. So, the system is profitable only if $C_0 > 625$. **Fig. 5** shows the behavior of profit with respect to revenue per unit up time (C_1) for different values of cost per visit of the repairman (C_3). It can be concluded from the graph that the profit decreases with the increase in values of C_1 and has lower values for higher values of C_3. It is also observed from the graph that for $C_3 = 100$, the profit is positive or zero or negative according as $C_1 <$ or $= > 1180$. So, the system is profitable only if $C_0 < 1180$. For $C_3 = 6100$, the profit is positive or zero or negative according as $C_1 <$ or $= > 1200$. So, the system is profitable only if $C_1 < 1200$. For $C_3 = 12100$, the profit is positive or zero or negative according as $C_1 <$ or $= > 1280$. So, the system is profitable only if $C_1 < 1280$.

4.1. Measures of Effectiveness
4.1.1. Mean Time to System Failure

To determine the Mean Time to System Failure (MTSF) of the system, we regard the failed states as absorbing states. By probabilistic arguments, we obtain the following recursive relations for $\phi_i(t)$:
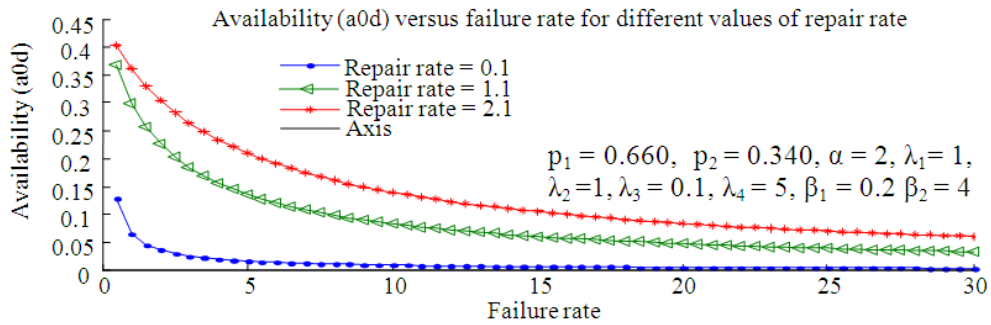
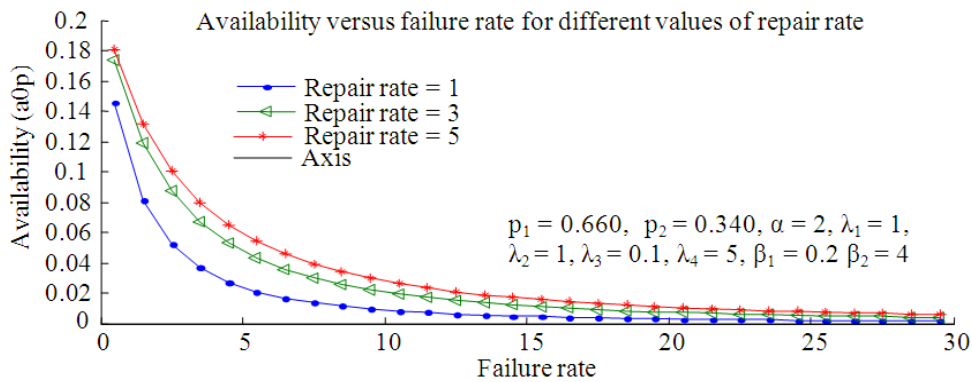**Fig. 2.** Availability versus Failure rate when demand is not less than production for different

Fig. 3. Availability versus Failure rate when demand is less than production for different values of repair rate
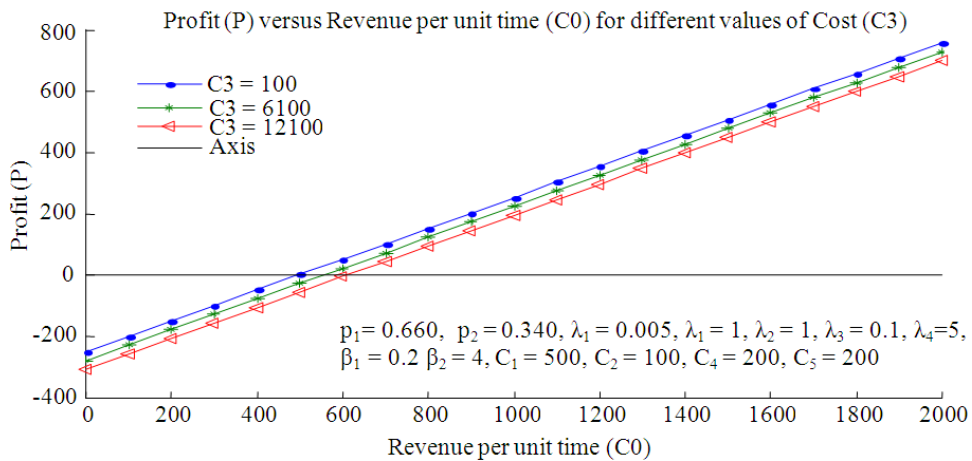

Fig. 4. Profit (P) versus Revenue per unit time (C0) for different values of Cost (C3)

$\phi_0(t) = Q_{01}(t) \text{\$} \phi_1(t) + Q_{02}(t) + Q_{03}(t) \text{\$} \phi_3(t)$
$\phi_1(t) = Q_{10}(t) \text{\$} \phi_0(t) + Q_{14}(t) + Q_{15}(t) \text{\$} \phi_5(t)$
$\phi_3(t) = Q_{30}(t) \text{\$} \phi_0(t)$
$\phi_5(t) = Q_{50}(t) \text{\$} \phi_0(t)$

Taking Laplace-Steltjes Transform (LST) of these relations and solving them for $\phi_0{}^{**}(s)$. The Mean Time to System Failure (MTSF) when the system starts from the state '0'is:
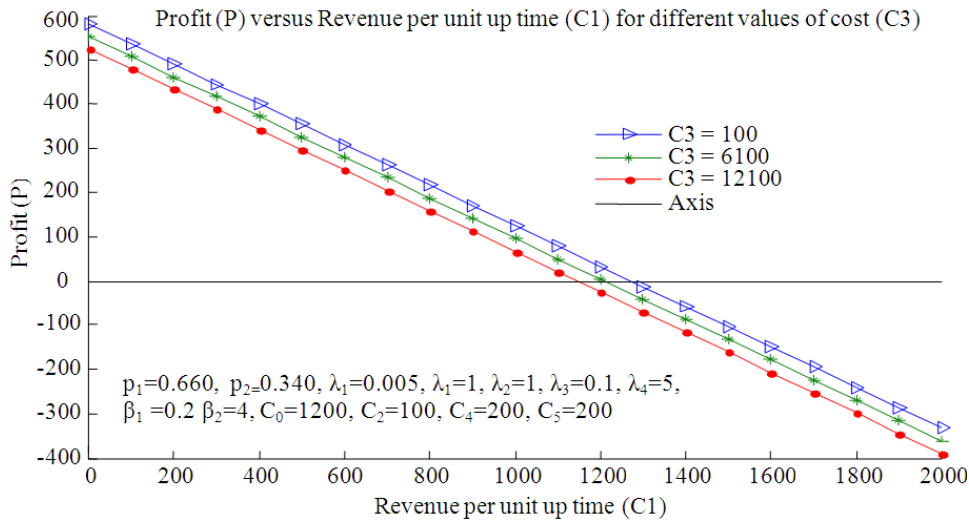
158

Fig. 5. Profit (P) versus Revenue per unit up time (C1) for different values of cost (C3)

$$MTSF = \lim_{s \to 0}(1 - \phi_0 **(s))/s = N/D$$

where, $N = \mu_0 + p_{01}\mu_1 + p_{03}\mu_3 + p_{01}p_{15}\mu_5$ and $D = 1 - p_{01}p_{14} - p_{03}$.

4.2. Availability Analysis when Demand is not less than Production

Using the arguments of the theory of the regeneration process, the availability $A_i^d(t)$ is seen to satisfy the following recursive relations:

$$A_0^d(t) = M_0(t) + q_{01}(t) \, \textcircled{c}A_1^d(t) + q_{02}(t) \, \textcircled{c}A_2^d(t) + q_{03}(t) \, \textcircled{c}A_3^d(t)$$
$$A_1^d(t) = q_{10}(t) \, \textcircled{c}A_0^d(t) + q_{14}(t) \, \textcircled{c}A_4^d(t) + q_{15}(t) \, \textcircled{c}A_5^d(t)$$
$$A_2^d(t) = q_{20}(t) \, \textcircled{c} \, A_0^d(t)$$
$$A_3^d(t) = q_{30}(t) \, \textcircled{c} \, A_0^d(t)$$
$$A_4^d(t) = q_{40}(t) \, \textcircled{c}A_0^d(t) + q_{41}(t) \, \textcircled{c}A_1^d(t)$$
$$A_5^d(t) = q_{50}(t) \, \textcircled{c} \, A_0^d(t)$$
$$\text{Where} = M_0(t) = e^{-(\lambda + \lambda_1 + \beta_1)t}$$

Taking Laplace transforms of the above equations and solving them for $A_0*(s)$. The availability of the system, in steady-state, is given by:

$$A_0^d = \lim_{s \to 0}(sA_0*(s)) = N_1/D_1$$

where, $N_1 = (1 - p_{14}p_{41})\mu_0$ and $D_1 = \mu_0 + p_{01}\mu_1 + p_{02}(1 - p_{14}p_{41})\mu_2 + p_{03}(1 - p_{14}p_{41})\mu_3 + (p_{01}p_{14})\mu_4 + (p_{01}p_{15})\mu_5$.

4.3. Availability Analysis when Demand is less than Production

Proceeding in the similar fashion as in 5.2, the availability of the system, in steady-state, is given by:

$$A_0^p = \lim_{s \to 0}(sA_0*(s)) = N_2/D_1$$

where, $N_2 = p_{01}\mu_1$ and D_1 is already specified.

4.4. Busy Period Analysis of the Repairman

By probabilistic arguments, the total fraction of the time for which the system is under repair of the ordinary repairman, in steady-state, is given by:

$$B_0^0 = \lim_{s \to 0}(sB_0^0*(s)) = N_3/D_1$$

where, $N_3 = p_{02}(1 - p_{14}p_{41})\mu_2 + p_{01}p_{14}\mu_4$ and D_1 is already specified.

4.5. Expected Number of Visits by the Repairman

The expected number of visits per unit time by the ordinary repairman is given by:

$$V_0^0 = \lim_{s \to 0}(sV_0^0*(s)) = N_4/D_1$$

where, $N_4 = p_{02}(1 - p_{14}p_{41}) + p_{01}p_{14}$ and D_1 is already specified.

4.6. Expected Number of Scheduled Maintenances

In steady-state, the expected number of scheduled maintenances per unit time is given by:

$$SM_0^0 = \lim_{s \to 0}(sSM_0^0 * (s)) = N_5 / D_1$$

where, $N_5 = p_{03}p_{30} (1\text{-}p_{14}p_{41})$ and D_1 is already specified.

4.7. Expected Down Time

The total fraction of the time for which the system is in down state given by:

$$DT_0^0 = \lim_{s \to 0}(sDT_0^0 * (s)) = N_5 / D_1$$

where, $N_6 = (p_{01}p_{15})\mu_5$ and D_1 is already specified.

The values of various measures of the system effectiveness are obtained as:

- Mean Time to System Failure (MTSF) = 1.9721
- The Steady State Availability Analysis when demand is not less than production (A_0^d) = 0.5048
- The Steady State Availability Analysis when demand is less than production (A_0^p) = 0.4576
- Busy period of the repairman for repair at t = 0 (B_0^0) = 0.0024
- Expected number of visits by the repairman at t = 0(V_0^0) = 0.0048
- Expected Number of scheduled maintenance at t = 0(SM_0^0) = 0.1010
- Expected down time at t = 0(DT_0^0) = 0.0092

4.8. Cost-Benefit Analysis

Profit (P) = C_0 A_0^d - C_1 A_0^p -C_2 B_0^0 -C_3 V_0^0 -C_4 SM_0^0 - $C_5 DT_0^0$:

C_0 = Revenue per unit up time when demand is not less than production
C_1 = Revenue per unit up time when demand is less than production
C_2 = Cost per unit up time for engaging the repairman for repair
C_3 = Cost per visit of the repairman
C_4 = Cost of scheduled maintenance per unit time. (All costs are in Indian rupee)
C_5 = Loss per unit time during the system remains down

5. CONCLUSION

From the interpretations as made above through various graphs, we can conclude that cut off points for various rates/probabilities/revenue per unit up time/costs can be obtained which help in deciding the upper/lower acceptable values of rates/costs so that the system is profitable. That is, the upper limit of the failure rate can be obtained, the lower value of the revenue per unit up time when demand is greater than or equal to production can be obtained on the basis of which the company can fix the price of the product manufactured by the company so that the system gives the positive profit. The upper/lower limits of various other rates/costs can be obtained. Obtaining such values, various suggestions can be given to the company using such systems.

6. REFERENCES

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