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Correlation Between Dispersion and Mean Toassess Healthcare Servicee Fficiency

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ABSTRACT

Motivation for this research work started while helping a hospital administrator to assess whether patient oriented activity duration, X≥0 is reflecting the service's efficiency? Higher value of the sample mean duration, X implies lesser productivity in the hospital and more healthcare cost. Likewise, larger value of sample dispersion, s_x^2 in the service durations is an indicator of lesser reliability and inefficiency. Of course, the dispersion, s_x^2 in a healthcare hospital operation could be due to diverse medical complications among patients or operational inefficiency. Assuming that it is not the diverse medical complications of patients, how should the pertinent information from data be extracted, quantified and interpreted to address inefficient operation? This is the problem statement for discussion in this article. To be specific, in an inefficient hospital operation, the sample dispersion and mean of service durations are likely to be highly correlated. Their correlation is a clue to identify an inefficient operation of a hospital. To compute the correlation, currently there is no appropriate formula in the literature. The aim of this article is, therefore, to derive a working formula to compute the correlation between sample dispersion and mean. The dispersion is too valuable statistical measure to quickly dispense, not only in healthcare operations but also in engineering, economics, business, social or sport applications. The approach starts first in quantifying a general relationship between the dispersion and mean in a given data. This relationship might range from a linear to a quadratic, cubic or higher degree. Suppose that the dispersion, σ^2 is a function, $f(\mu)$of the mean, μ of patient oriented activity durations. Specific functionality depends on the frequency pattern of the data. The tangent at a locus of their relationship curve is either declining or inclining line with an angle θ whose cosine value is indeed the correlation between the mean, x and dispersion, s_x^2. An expression to compute the angle is nowhere seen in the literature. Therefore, this article derives a general expression based on geometric concepts and then obtains specific formula for several count and continuous distributions. These expressions are foundations for further data analyses. To initiate, promote or maintain an efficient service operation for patients in a hospital, practical strategies have to be formulated based on the cluein the form of correlation value. For this purpose, a one-to-one relationship between sample dispersion and mean could be utilized to improve the service efficiency. In this process, a formula is developed to check whether the model parameters are orthogonal. The curvature and the shifting angle in the relationship between dispersion and mean are captured when the mean changes one unit. Both Poisson and exponential distributions are illustrated to comprehend the concepts and the derived expressions of this article. Efficient healthcare service is a necessity not only in USA but also in other nations because of an escalating demand by medical tourists in this era of globalized medical treatment. A reformation to the entire healthcare field could be achievable with the help of biostatical concepts and tools. To extract and comprehend pertinent data information in the patient oriented activity durations, the correlation is a tool. The data information holds the key to make the much needed reformation and operational efficiency. This article illustrates that the correlation between the data mean and dispersion provides clues. The correlation helps to assess healthcare service efficiency as it is demonstrated in this article with data. Similar applications occur in engineering, business and science fields.

Keywords: Data Mean and Dispersion, Curvature, Healthcare Management, Shifting Angle

1. INTRODUCTION

The dispersion, σ^2 is well connected to the mean, μ in probability distributions (Evans *et al*., 2000). Any one of these could be an underlying model for the data to be analyzed and interpreted. The relation, $f(\mu)$ between the dispersion and mean is useful in data analysis. This article demonstrates it.

Table 1. Summary of correlation, $r_{\bar{x},s^2}$ for distributions.

Distribution	Correlation, $r_{\bar{x},s^2}$
Beta	$\sqrt{\frac{\{(1-\bar{x})\bar{x}\}^2}{(1-2\bar{x})^2 s^2+\{(1-\bar{x})\bar{x}\}^2}}$
Beta binomial; β≥1	$\sqrt{\frac{\{\frac{(n-2\bar{x})-\bar{x}}{\bar{x}(n-\bar{x})}+\frac{(n+1)\beta+2\bar{x}}{(\beta+\bar{x})(n\beta+\bar{x})}\}^{-2}}{s^2+\{\frac{(n-2\bar{x})-\bar{x}}{\bar{x}(n-\bar{x})}+\frac{(n+1)\beta+2\bar{x}}{(\beta+\bar{x})(n\beta+\bar{x})}\}^{-2}}};\beta\ge 1$
Binomial (n=1 for Bernoulli)	$\sqrt{\frac{\{(n-\bar{x})\bar{x}\}^2}{(n-2\bar{x})^2 s^2+\{(n-\bar{x})\bar{x}\}^2}}$
Consul; β≥1	$\sqrt{\frac{\{\frac{2\beta\bar{x}-1}{\beta\bar{x}^2}+\frac{1}{\bar{x}-1}\}^{-2}}{s^2+\{\frac{2\beta\bar{x}-1}{\beta\bar{x}^2}+\frac{1}{\bar{x}-1}\}^{-2}}};\beta\ge 1$
Discrete uniform	$\sqrt{\frac{\{\bar{x}(\bar{x}+1)\}^2}{(2\bar{x}+1)^2 s^2+\{\bar{x}(\bar{x}+1)\}^2}}$
Erlang	$\sqrt{\frac{\bar{x}^2}{4s^2+\bar{x}^2}}$
F; β≥1	$\sqrt{\frac{\{\bar{x}[\beta(\bar{x}-1)+2]\}^2}{\{3\beta(\bar{x}-1)+\beta\bar{x}+6\}^2 s^2+\{\bar{x}[\beta(\bar{x}-1)+2]\}^2}};\beta\ge 1$
Gamma (β≥1 is exponential)	$\sqrt{\frac{\beta^2\bar{x}^2}{4s^2+\beta^2\bar{x}^2}};\beta\ge 1$
Geeta; β≥1	$\sqrt{\frac{\{\frac{2\bar{x}-1}{\bar{x}(\bar{x}-1)}+\frac{\beta}{\beta\bar{x}-1}\}^{-2}}{s^2+\{\frac{2\bar{x}-1}{\bar{x}(\bar{x}-1)}+\frac{\beta}{\beta\bar{x}-1}\}^{-2}}};\beta\ge 1$
Generalized binomial; β≥0	$\sqrt{\frac{\{n+2[\beta-1]\bar{x}\}^{-2}}{\{n+[\beta-1]\bar{x}\}^2\bar{x}^2 s^2+\{n+2[\beta-1]\bar{x}\}^{-2}}};\beta\ge 0$
Generalized Katz; β≥0	$\sqrt{\frac{\{\bar{x}(\alpha+\beta\bar{x})\}^2}{\{3\alpha+4\beta\bar{x}\}^2 s^2+\{\bar{x}(\alpha+\beta\bar{x})\}^2}};\alpha,\beta\ge 0$
Geometric	$\sqrt{\frac{\{\bar{x}(\bar{x}-1)\}^2}{(2\bar{x}+1)^2 s^2+\{\bar{x}(\bar{x}-1)\}^2}}$
Hyper geometric; N≥1	$\sqrt{\frac{\{\frac{n-2\bar{x}}{\bar{x}(n-2\bar{x})}+\frac{(n+1)N-2\bar{x}}{(nN-\bar{x})(N-\bar{x})}\}^{-2}}{s^2+\{\frac{n-2\bar{x}}{\bar{x}(n-2\bar{x})}+\frac{(n+1)N-2\bar{x}}{(nN-\bar{x})(N-\bar{x})}\}^{-2}}};N\ge 1$
Intervened Poisson (β = 0 for positive Poisson)	$\sqrt{\frac{\{\bar{x}(\bar{x}-\beta\theta)\}^2}{(\bar{x}+\beta\theta)^2 s^2+\{\bar{x}(\bar{x}-\beta\theta)\}^2}};\beta\ge 0$
Inverse Gaussian; β≥0	$\sqrt{\frac{\{\bar{x}(\beta+\bar{x})\}^2}{(\beta+2\bar{x})^2 s^2+\{\bar{x}(\beta+\bar{x})\}^2}};\beta\ge 0$
Lagrangian Poisson; β≥0	$\sqrt{\frac{\{\bar{x}\}^2}{\{1-\beta\}^2 s^2+\{\bar{x}\}^2}};\beta\ge 0$
Lognormal; β≥0	$\sqrt{\frac{\{\bar{x}(\bar{x}+\beta)(\bar{x}-\beta)\}^2}{4(2\bar{x}^2-\beta^2)^2 s^2+\{\bar{x}(\bar{x}+\beta)(\bar{x}-\beta)\}^2}};\beta\ge 0$

Table 1. Continue

Lomax; β≥0	$\sqrt{\frac{\{\bar{x}(1-\beta^2\bar{x}^2)\}^2}{4(1-2\beta^2\bar{x}^2)^2 s^2+\{\bar{x}(1-\beta^2\bar{x}^2)\}^2}};\beta\ge 0$
Neyman Type A; β≥0	$\sqrt{\frac{\{\bar{x}(\beta+2\bar{x})\}^2}{\{\beta+2\bar{x}\}^2 s^2+\{\bar{x}(\beta+2\bar{x})\}^2}};\beta\ge 0$
Noncentral chi squared (for chi-squaredβ = 0)	$\sqrt{\frac{(\bar{x}+\beta)^2}{s^2+(\bar{x}+\beta)^2}};\beta\ge 0$
Pareto; β≥0	$\sqrt{\frac{\{\bar{x}(\bar{x}-\beta)(2\beta-\bar{x})\}^2}{4(3\beta\bar{x}-\beta^2-\bar{x}^2)^2 s^2+\{\bar{x}(\bar{x}-\beta)(2\beta-\bar{x})\}^2}};\beta\ge 0$
Poisson	$\sqrt{\frac{\bar{x}^2}{s^2+\bar{x}^2}}$
Power function; β≥0	$\sqrt{\frac{\{\bar{x}(\beta-\bar{x})\}^2}{(2\beta-3\bar{x})^2 s^2+\{\bar{x}(\beta-\bar{x})\}^2}};\beta\ge 0$
Random walk; β≥0	$\sqrt{\frac{\{\frac{1}{\bar{x}}+\frac{2\bar{x}}{\bar{x}^2-\beta^2}\}^{-2}}{s^2+\{\frac{1}{\bar{x}}+\frac{2\bar{x}}{\bar{x}^2-\beta^2}\}^{-2}}};\beta\ge 0$
Uniform; β≥0	$\sqrt{\frac{\{(\bar{x}-\beta)\}^2}{4s^2+\{(\bar{x}-\beta)\}^2}};\beta\ge 0$

Table 2. Curvature c and shifting angle ϕ for distributions

Distribution	$f(\bar{x})$	Curvature $c=f(\bar{x})\{[\partial_{\bar{x}}\ln f(\bar{x})]^2+[\partial^2_{\bar{x}\bar{x}}\ln f(\bar{x})]\}$	Shifting angle $\phi=\frac{c}{\{1+[f(\bar{x})\partial_{\bar{x}}\ln f(\bar{x})]^2\}}$
Beta; β≥0	$\frac{\bar{x}^2(1-\bar{x})}{(\beta+\bar{x})};\beta\ge 0$	$6\{\frac{(1-\bar{x})}{(\beta+\bar{x})}-\{\frac{\bar{x}}{\beta+\bar{x}})^2\}$	$\frac{c}{\{1+[\frac{\bar{x}(1-\bar{x})}{(\beta+\bar{x})}(2+\frac{\bar{x}}{\beta+\bar{x}})]^2\}}$
Beta binomial; β≥0	$\bar{x}(1-\frac{\bar{x}}{n})(\frac{\beta+\bar{x}}{n\beta+\bar{x}});\beta>0$	$\frac{2}{(n\beta+\bar{x})}\{n-\beta-3\bar{x}+\frac{(\beta+\bar{x})([n+2]\bar{x})-n-\bar{x}^2)-\bar{x}(n-\bar{x})}{(n\beta+\bar{x})}\}$	$\frac{c}{\{1+[\frac{(\beta+\bar{x})(n-2\bar{x})+\frac{\beta\bar{x}(n-\bar{x})(n-1)}{(n\beta+\bar{x})}}{(n\beta+\bar{x})}]^2\}}$
Binomial (n=1 for Bernoulli)	$\bar{x}(1-\frac{\bar{x}}{n})$	$-\frac{2}{n}$	$\phi=\frac{c}{\{1+(1-\bar{x}/n)^2\}}$
Consul; β≥0	$\frac{\bar{x}(1-\bar{x})(1+[\beta-1]\bar{x})}{\beta}$	$\frac{2(1+[\beta-1][3\bar{x}-1])}{\beta}$	$\frac{c}{\{1+[\frac{(1+[\beta-1]\bar{x})(2\bar{x}-1)+(\beta-1)\bar{x}^2}{\beta}]^2\}}$
Discrete uniform	$\frac{\bar{x}(\bar{x}+1)}{3}$	$\frac{2}{3}$	$\frac{c}{\{1+[\frac{2\bar{x}+1}{3}]^2\}}$
Erlang, β≥0	$\frac{\bar{x}^2}{\beta}$	$\frac{2}{\beta}$	$\frac{c}{\{1+\frac{4\bar{x}^2}{\beta}\}}$
F; β≥0	$\bar{x}^3(\frac{\beta[\bar{x}-1]+2}{2})$	$3\bar{x}(\beta[2\bar{x}-1]+2)$	$\frac{c}{\{1+\frac{\bar{x}^4[\beta\bar{x}-3\beta+6]^2}{4}\}}$
Gamma (β≥0 for exponential)	$\frac{\bar{x}^{-2}}{\beta}$	$\frac{2}{\beta}$	$\frac{c}{\{1+\frac{4\bar{x}^2}{\beta}\}}$

Table 2. Continue

Geeta; β≥0	$\bar{x}(2\beta\bar{x}-1)(\bar{x}-1)$	$2\beta(3\bar{x}-2)$	$\frac{c}{\{1+[(\beta\bar{x}-1)(\bar{x}-1)+\beta\bar{x}(\bar{x}-1)+\bar{x}(\beta\bar{x}-1)]^2\}}$
Generalized binomial(β = 0 for binomial,β = 1 for inverse binomial)	$\bar{x}[\frac{n+(\beta-1)\bar{x}}{n}]$	$\frac{2(\beta-1)}{n}$	$\frac{c}{1+[\frac{n+2(\beta-1)\bar{x}}{n}]^2}$
Generalized Katz α>0;β≥0	$\frac{(1-\beta)\bar{x}^3}{\alpha^2};\alpha,\beta>0$	$\frac{6\beta\bar{x}^2}{\alpha^2}$	$\phi=\frac{c}{\{1+[\frac{\bar{x}^2(3\alpha+4\beta\bar{x})}{\alpha^2}]^2\}}$
Geometric	$\bar{x}(\bar{x}+1)$	2	$\frac{c}{\{1+[1+2\bar{x}]^2\}}$
Hyper geometric	$\bar{x}(1-\frac{\bar{x}}{n})(\frac{N-n}{N-1})$	$-\frac{2(N-n)}{n(N-1)}$	$\phi=\frac{c}{\{1+[\frac{(N-n)(n-2\bar{x})}{n(N-1)}]^2\}}$
Intervened Poisson (β = 0 for positive poisson)	$\bar{x}-[\bar{x}-(1+\beta)\theta][\bar{x}-\beta\theta]$	2	$\frac{c}{\{1+[1-2(\bar{x}-\beta\theta)+\theta]^2\}}$
Inverse binomial; β>0	$\frac{\bar{x}(\bar{x}+\beta)}{\beta}$	$\frac{2}{\beta}$	$\frac{c}{\{1+[1+\frac{2\bar{x}}{\beta}]^2\}}$
Inverse Gaussian; β≥0	$\frac{\bar{x}^3}{\beta}$	$\frac{6\bar{x}}{\beta}$	$\frac{c}{\{1+\frac{9\bar{x}^4}{\beta^2}\}}$
Lagrangian Poisson; -1<β<1	$\frac{\bar{x}}{(1-\beta)^2}$	0	0
Lognormal; β>0	$\frac{\bar{x}^2(\bar{x}^2-\beta^2)}{\beta^2}$	$2(\frac{6\bar{x}^2}{\beta^2}-1)$	$\frac{c}{\{1+4\bar{x}^2[\frac{2\bar{x}^2}{\beta^2}-1]^2\}}$
Lomax; β≥0	$(\frac{1+\beta\bar{x}}{1-\beta\bar{x}})\bar{x}^2$	$2(\frac{[1-2\beta\bar{x}][1+2\beta\bar{x}]}{[1-\beta\bar{x}]^2})$	$\frac{c}{\{1+[\frac{2\bar{x}(1-2\beta^2\bar{x}^2)}{(1-\beta\bar{x})^2}]^2\}}$
Neyman Type A, β>0	$\frac{\bar{x}(\beta+\bar{x})}{\beta}$	$\frac{2}{\beta}$	$\frac{c}{\{1+[\frac{\beta+2\bar{x}}{\beta}]^2\}}$
Noncentral chi squared (for chi-squared β = 0	$2(\bar{x}+\beta);\beta\geq 0$	0	0
Pareto; β≥0	$\frac{\bar{x}(\bar{x}-\beta)^2}{(2\beta-\bar{x})}$	$\frac{2(\bar{x}-\beta)^2}{(2\beta-\bar{x})}\{1+2(\frac{\beta}{2\beta-\bar{x}})^2\}$	$\frac{c}{\{1+[\frac{2(\bar{x}-\beta)}{(2\beta-\bar{x})^2}(\beta\bar{x}-[\bar{x}-\beta]^2)]^2\}}$
Poisson	$\bar{x}$	0	0
Power function; β>0	$\frac{\bar{x}^2}{\beta}(\frac{\beta-\bar{x}}{2\beta-\bar{x}})$	$\frac{2(2\beta-5\bar{x})}{(2\beta-\bar{x})^2}$	$\frac{c}{\{1+[\frac{(2\beta-3\bar{x})\bar{x}}{\beta(2\beta-\bar{x})}]^2\}}$
Random walk; β>0	$\bar{x}[(\frac{\bar{x}}{\beta})^2-1]$	$\frac{4\bar{x}}{\beta^2}$	$\frac{c}{1+[3(\frac{\bar{x}}{\beta})^2-1]^2}$
Uniform; β>0	$\frac{(\bar{x}-\beta)^2}{3}$	$\frac{2}{3}$	$\frac{c}{\{1+\frac{4(\bar{x}-\beta)^2}{9}\}}$

In fact, the data mean, $\bar{x}$ and dispersion, s_x^2 are correlated. Their correlation is indicated by $r_{\bar{x},s^2}$. Neither a textbook nor a journal article provides explicit expression to compute their correlation or how to use it in data analysis. This article derives geometrically an expression for the correlation. Specific expressions to compute the correlation for specific count and continuous probability distributions are obtained and displayed in **Table 1**. Applied data analysts and healthcare decision makers could compute and use the correlation to assess whether the healthcare services are efficiently done after the selection of an underlying distribution for the data.

Also developed in this article are a general expression for the curvature (c) in the relation between dispersion and mean and the shifting angle, ϕ of the curvature. The curvature is an interesting concept. For example, the curvature of a circle is reciprocal to its radius and consequently, the smaller circle bends quickly with higher curvature. However, particular formulas are obtained for potential count or continuous probability distributions which might be appropriate for a given data. The curvature and shifting angle (**Table 2**) provide a paradigm in healthcare service efficiency assessment as illustrated.

2. GEOMETRIC DERIVATION OF CORRELATION BETWEEN DISPERSION AND MEAN

Suppose that a random data $x_1,x_2,\ldots..x_n$ is drawn from a probability distribution $g(x|\mu,\sigma^2)$ which governs the chance mechanism of the collected data, where μ and $\sigma_2$are mean and dispersion parameter respectively. The data mean, $\bar{x}$ and dispersion, s_x^2 are natural (in fact, maximum likelihood estimator) of the mean and dispersion parameter. These estimators are correlated. There is no formula in the literature to compute the correlation. To visualize their correlation, consider the configuration of the relation, σ^2 = f between dispersion, σ^2 and mean, μ in **Fig. 1**.

In the case of normal distribution, the function $f(\mu)$ does not exist as dispersion, σ^2 and mean, μ are disconnected and independent. In other symmetric non-normal distributions, the dispersion, $f(\mu)$ and mean, μ parameters could be disconnected but not necessarily independent. Their data counterparts s^2 and $\bar{x}$ could be correlated but there is no formula in the literature to compute it. In non-symmetric non-normal count or continuous distributions in **Table 1**, the function f (μ) does exist as dispersion, σ^2 and mean, μ are connected and dependent. The non-orthogonality between them is captured by Cosθ where the angle θ is acute or obtuse depending on whether $f(\mu)<$ or $f(\mu)>\mu$.

For an example, when $f(\mu) = \mu$ as in the Poisson distribution, the angle is 45°C. To derive a general formula, the geometric view is helpful. To derive an expression for Cosθ, note that the opposite side of the angle has length f (μ). To find the hypotenuse, why not first obtain the tangent's equation and it is $y-f(\mu) = (x-\mu)\partial_\mu f(\mu)$ where the notation $\partial_m f(m)$ denotes the derivative with respect to m. This tangent crosses the horizontal axis at a location $(\mu_0,0)$ and consequently the horizontal distance between the locations $(\mu_0,0)$ and$(\mu_0,0)$ is $\mu-\mu_0 = [\partial_\mu \text{In } f(\mu)]^{-1}$. Hence, the cosine angle is the elevation of the dispersion with reference to the mean axis and it is:

$$\theta = \cos^{-1}[(1+\sigma^2[\partial_\mu \ln f(\mu)]^2)^{-1/2}] \quad (1)$$

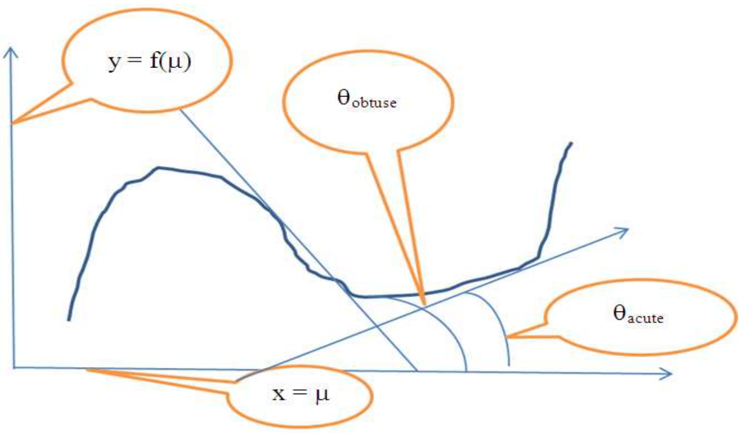

Fig. 1. Dispersion $y = f(\mu)$ in terms of mean $x = \mu$ with angle θ_{acute} or θ_{obtuse}

which, is dynamically shifting, depending on the mean location and the oscillation of the curve f (μ). Consequently, an expression could be derived for the finite correlation, $r_{\bar{x},s^2}$ as in (2) between the data dispersion, s_x^2 and mean, $\bar{x}$. The correlation could be computed for a given data using (2). That is:

$$\left|r_{\bar{x},s^2}\right| = \left|\hat{\cos}\theta\right| = (1 + s^2[\partial_{\bar{x}} \ln f(\bar{x})]^2)^{-1/2} \quad (2)$$

The curvature, $c = \partial^2_{\bar{x}\bar{x}} f(\bar{x})$ of the relation connecting dispersion and mean could be derived using (2) as follows. Note that:

$$c = f(\bar{x})\{[\partial_{\bar{x}} \ln f(\bar{x})]^2 + \sqrt{\frac{s^2}{(s^2 - r^2_{\bar{x},s^2})^3}}\partial_{\bar{x}} r_{\bar{x},s^2}\} = f(\bar{x})\{[\partial_{\bar{x}} \ln f(\bar{x})]^2 + [\partial^2_{\bar{x}\bar{x}} \ln f(\bar{x})]\}. \quad (3)$$

The shifting angle, $\phi = \frac{\partial^2_{\bar{x}\bar{x}} f(\bar{x})}{1 + [\partial_{\bar{x}} f(\bar{x})]^2}$ is then:

$$\phi = \frac{f(\bar{x})\{[\partial_{\bar{x}} \ln f(\bar{x})]^2 + [\partial^2_{\bar{x}\bar{x}} \ln f(\bar{x})]\}}{\{1 + [f(\bar{x})\partial_{\bar{x}} \ln f(\bar{x})]^2\}}. \quad (4)$$

These results are explained with Poisson example in the count category and exponential example in the continuous category below.

3. POISSSON EXAMPLE

For an example in count distributions category, consider the Poisson distribution $f(x|\lambda) = e^{-\lambda}\lambda^x|x!;x = 0,1,2,\ldots;\lambda>0$ in which the dispersion, σ^2 and mean μ are respectively λ. Hence, $f(\mu) = \mu$ and the non-orthogonality between the dispersion and mean, according to (1), is $\theta = \cos^{-1}[\sqrt{\frac{\lambda}{1+\lambda}}]$, an acute angle. The correlation, according to (2), between the Poisson data dispersion and mean is $\left|r_{\bar{x},s^2}\right| = \sqrt{\frac{s^2}{s^2 + \bar{x}^2}}$. The curvature in the Poisson distribution, according to (3), is c = 0 and the shifting angle of the Poisson curvature, according to (4) is $\phi = 0$.

4. EXPONENTIAL EXAMPLE

For an example in continuous distributions category, consider the exponential distribution $f(x|\lambda) = \frac{1}{\lambda}e^{-\frac{x}{\lambda}};x > 0;\lambda > 0$ in which the dispersion, σ^2 and mean μ are respectively λ^2 and λ. Note that $f(\mu) = \mu^2$ and the non-orthogonality between the dispersion and mean, according to (1), is $\theta = \cos^{-1}[2\sqrt{\frac{\lambda}{5}}]$, an acute, 45^0 or obtuse angle depending on $\lambda<1, \lambda = 1$ or $\lambda >1$ respectively. The correlation, according to (2), between the exponential distribution data dispersion and mean is $\left|r_{\bar{x},s^2}\right| = \sqrt{\frac{s^2}{s^2 + 4\bar{x}^2}}$, where $r_{\bar{x},s^2}$ is less than one. The curvature in the exponential distribution, according to (3), is c = 2 and the shifting angle of the exponential distribution curvature, according to (4) is $\phi = (\frac{2}{1 + 4\bar{x}^2})$.

The statistical community and data analysts in applied disciplines have expressed interest in dispersion issues and have been discussing it as under, equal or over dispersion (Lindsey, 1999). However, for the sake of users in applied disciplines, specific expressions are derived and displayed in Section 3 and are illustrated with healthcare data in Section 4.

5. EXPRESSIONS FOR COUNT AND CONTINUOUS DISTRIBUTIONS

Particular expression of the correlation $r_{\bar{x},s^2}$ using (2), the curvature c using (3) and the shifting angle, ϕ of the curvature using (4) for several distributions are derived and displayed alphabetically in **Table 1 and 2**, where β denotes a nuisance parameter.

6. HEALTHCARE APPLICATION: ARE PATIENTS EFFICIENTLY SERVED?

The results of Section 3 are illustrated using patient service activities data in Ozcan (2005) and other data.

First, consider the data in **Table 3** about the number, X of emergency pickups from home in Austin, Texas over the weekdays of the months in 2011. Clearly, the pickups are Poisson event because of rarity. The Poisson correlation, $\sqrt{\frac{\bar{x}^2}{s^2 + \bar{x}^2}}$ is displayed with the data. Notice that all correlations between the dispersion and mean in the data are above 0.5. There is no curvature for Poisson data and hence no shifting angle.

Table 3. Correlation, $r_{\bar{x},s^2}$ in Poisson data

Year 2011	Mon	Tue	Wed	Thurs	Fri	Sat	Sun	$\bar{x}$	s^2	$r_{\bar{x},s^2}$
January	12.00	11.00	12.00	9.00	13.00	12.00	9.00	11.140	2.48	0.90
February	9.00	7.00	6.00	8.00	7.00	9.00	5.00	9.214	2.24	0.90
March	8.00	8.00	7.00	9.00	19.00	11.00	6.00	8.500	19.20	0.55
April	4.00	3.00	4.00	5.00	6.00	4.00	8.00	7.286	2.81	0.85
May	8.00	7.00	6.00	9.00	10.00	7.00	5.00	6.143	2.95	0.82
June	18.00	21.00	18.00	19.00	21.00	20.00	17.00	13.290	2.48	0.92
July	5.00	6.00	5.00	7.00	8.00	6.00	9.00	12.860	2.29	0.92
August	8.00	5.00	6.00	8.00	9.00	10.00	12.00	7.429	5.57	0.76
September	4.00	3.00	3.00	5.00	6.00	5.00	12.00	6.857	9.62	0.65
October	6.00	5.00	5.00	4.00	6.00	7.00	11.00	5.857	5.24	0.73
November	9.00	11.00	12.00	11.00	9.00	7.00	4.00	7.643	7.67	0.71
December	12.00	9.00	12.00	10.00	19.00	11.00	18.00	11.000	15.30	0.65
$\bar{x}$	8.58	8.00	8.00	8.67	11.10	9.08	9.67			
s^2	15.90	23.80	20.00	15.20	31.00	18.30	20.80			
$r_{\bar{x},s^2}$	0.71	0.57	0.62	0.73	0.59	0.69	0.67			

ambulatory pickups in a hospital at Austin, Texas during 2011 (Poisson data)

Table 4. Correlation, $r_{\bar{x},s^2}$ in exponential data about time spent in a hospital by eleven patients

Patient Activity	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P11	$\bar{x}$	s^2	$r_{\bar{x},s^2}$	ϕ
Registration	3.00	6.00	4.00	8.00	4.00	5.00	4.00	6.00	4.00	6.00	4.00	4.19	2.09	0.86	0.02
Co Payment	7.00	9.00	11.00	8.00	12.00	9.00	6.00	11.00	9.00	12.00	10.00	9.45	3.87	0.92	0.01
Waiting For Nurse	12.00	15.00	17.00	12.00	11.00	17.00	12.00	19.00	12.00	20.00	18.00	15.00	11.00	0.19	0.00
Vital Signs	9.00	8.00	11.00	12.00	9.00	8.00	10.00	12.00	8.00	12.00	11.00	.10	2.80	0.95	0.00
Waiting For Exam Room	12.00	15.00	12.00	14.00	21.00	18.00	11.00	16.00	9.00	14.00	18.00	14.50	12.50	0.90	0.00
Placement In Exam Room	3.00	5.00	4.00	6.00	3.00	5.00	3.00	6.00	5.00	4.00	7.00	4.64	1.85	0.86	0.02
Wait For Physician	10.00	17.00	21.00	11.00	13.00	15.00	14.00	12.00	19.00	15.00	9.00	14.20	14.00	0.88	0.00
Examination	18.00	15.00	19.00	22.00	18.00	12.00	19.00	21.00	16.00	21.00	17.00	18.00	8.60	0.95	0.00
Test Order	4.00	7.00	3.00	5.00	4.00	11.00	9.00	12.00	11.00	14.00	9.00	8.09	13.90	0.74	0.01
Referral Request	11.00	10.00	16.00	9.00	8.00	9.00	7.00	7.00	9.00	7.00	6.00	9.00	7.60	0.85	0.01
Follow up Entry	3.00	5.00	3.00	4.00	3.00	4.00	4.00	5.00	3.00	3.00	5.00	3.82	0.76	0.91	0.03
$\bar{x}$	8.36	10.20	11.00	10.10	9.64	10.30	9.00	11.50	9.55	11.60	10.40	10.10	1.01		
s^2	23.70	20.40	45.20	25.50	37.30	23.40	23.80	29.10	23.70	36.70	26.50	28.60	59.60		
$r_{\bar{x},s^2}$	0.65	0.75	0.63	0.71	0.62	0.73	0.68	0.73	0.70	0.69	0.71				
ϕ	0.01	0.00	0.00	0.00	0.01	0.00	0.01	0.00	0.01	0.00	0.00				

Ozcan (2005) Quantitative Methods in Health Care Management, New York: John Wiley Press (Hospital outpatient clinic efficiency) X = Minutes spent for patient (exponential data)

Table 5. Correlation, $r_{\bar{x},s^2}$ in binomial data (number of patients spending more than 10 minutes in activities)

Patient activity	X
Registration	0.00
Co Payment	5.00
Waiting for Nurse	11.00
Vital Signs	5.00
Waiting for Exam Room	10.00
Placement in Exam Room	0.00
Wait for Physician	9.00
Examination	11.00
Test Order	4.00
Referral Request	2.00
Follow up Entry	0.00
$\bar{x}$	5.18
s^2	19.80
$r_{\bar{x},s^2}$	0.98
ϕ	-0.01

Next, the data from Ozcan (2005) about the time, Y spent by patients in a hospital for several activities are considered. The exponential distribution is a natural choice for the data. The data and the exponential correlation, $\sqrt{\frac{\bar{x}^2}{4s^2+\bar{x}^2}}$ are displayed in **Table 4**. X = # patients spending more than 10 min in ten services.

Notice the exponential dispersion and average are highly correlated in all activities and for all patients implying that more dispersion occurs among the patients who spent more time. The curvature is 2 for the exponential data but the curve is shifting by an angle, $\frac{2}{\{1+4\bar{x}^2\}}$ that changes along with mean, $\bar{x}$.

Table 6. Correlation, $r_{\bar{x},s^2}$ in geometric data (# completed activities before one taking more than 10 min)

X = # activities	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P11
	2.00	2.00	1.00	2.00	1.00	2.00	2.00	1.00	2.00	1.00	2.00
	1.00	1.00	1.00	0.00	0.00	1.00	1.00	0.00	3.00	0.00	0.00
	2.00	1.00	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
	1.00	0.00	0.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
			0.00	0.00	0.00	0.00		1.00		1.00	2.00
			1.00					0.00		0.00	
			0.00					0.00		0.00	
			0.00								
			1.00								
$\bar{x}$	1.50	1.00	0.44	0.60	0.60	0.80	1.00	0.29	1.25	0.29	0.80
s^2	0.33	0.67	0.28	0.80	0.30	0.70	0.67	0.24	2.25	0.24	1.20
$r_{\bar{x},s^2}$	0.54	0.00	0.97	0.80	0.91	0.30	0.00	0.70	0.14	0.70	0.24
ϕ	0.12	0.20	0.44	0.34	0.34	0.26	0.20	0.58	0.15	0.58	0.26

X = # activities completed before an activity taking more than 10 min

Next, consider the number among n = 11patients spending more than 10 min in activities at a hospital and the data follow a binomial distribution. The correlation, $\sqrt{\frac{\{(n-\bar{x})\bar{x}\}^2}{(n-2\bar{x})^2 s^2 + \{(n-\bar{x})\bar{x}\}^2}}$ and shifting angle, $\phi = \frac{c}{\{1+(1-2\bar{x})^2\}}$ are shown in **Table 5**. The shifting angle of the curvature is less. The correlation between the binomial mean and dispersion is 0.97, a large amount. It means the dispersion increases along with mean spending time.

Next, consider geometric distributed number, Y of completed activities by n = 11 patients before experiencing an activity which took more than 10 min in a hospital as displayed in **Table 6**. The correlations, $\sqrt{\frac{\{\bar{x}(\bar{x}-1)\}^2}{(2\bar{x}+1)^2 s^2 + \{\bar{x}(\bar{x}-1)\}^2}}$ between the geometric mean and dispersion are low only for patients 2, 6, 7, 9 and 11 with the moderate shifting angle, $\frac{2}{\{1+[1+2\bar{x}]^2\}}$ of their curvature. The curvature of geometric distributed data is 2.

To be brief, other distributions are not illustrated. The data from other distributions could be similarly illustrated with appropriate healthcare management data.

7. CONCLUSION

The expression of this article to compute the correlation between sample dispersion and mean is a foundation to build a regression methodology for analyzing healthcare cost data which might follow any one of the distributions that are listed in **Table 1**. A regression methodology helps to identify or assess the importance of the predictors of the healthcare cost.

8. REFERENCES

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