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A Numerical Approach for Evaluating the Convergence-Confinement Curve of a Rock Tunnel Considering Hoek-Brown Strength Criterion

Pierpaolo Oreste

Department of Environmental, Land and Infrastructural Engineering, Politecnico di Torino (Italy)
Corso Duca degli Abruzzi 24, I-10129 Torino (Italy)

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E-mail: pierpaolo.oreste@polito.it

Abstract: The convergence-confinement method is frequently used for the analysis of the behavior of a tunnel and to define the support structures necessary to guarantee its stability. The reasons why it is used extensively, at least in the preliminary study phases of a tunnel are: The fundamental parameters of the problem (the diameter and depth of the tunnel, as well as the mechanical characteristics of the ground in which the tunnel is being excavated) can be considered in the calculation; the stresses and strains in the ground and the displacements of the tunnel wall can be obtained from the results of the calculation; the thickness of the plastic zone around the tunnel can be estimated by the method. The application of this method to rock masses, which are characterized by a non-linear strength criterion, has been limited to a restrictive number of works in the literature. In order to consider non-linear strength criteria, some simplifications are generally introduced in order to be able to obtain a simple expression of the stresses and strains in the rock mass and of the displacements of the tunnel wall. A numerical solution is presented in this study with which it is possible, in a simple way, to precisely describe the convergence-confinement curve of a tunnel excavated in rock masses, without the need of introducing any simplifying hypotheses for the description of the stress-strain relations in a plastic field.

Keywords: Tunnel Wall Displacement, Convergence-Confinement Curve, Rock-Support Interaction, Elasto-Plastic Behaviour, Deep Circulartunnel, Rock Mass, Hoek-Brown Strength Criterion

Introduction

The convergence-confinement method has the objective of describing the relationship between the internal applied pressure and the radial displacements of a tunnel wall (Rechsteiner and Lombardi, 1974; Ribacchi and Riccioni, 1977; Panet and Guenot, 1982; Lembo-Fazio and Ribacchi, 1986; AFTES, 1993; Peila and Oreste, 1995; Panet, 1995; Panet *et al*., 2001). Such a relationship is generally obtained through the use of simple equations for both the elastic and the elasto-plastic fields; the method allows the static conditions of the tunnel to be described, even in consideration of the support structures that have to be foreseen (Oreste, 2003a; 2003b; 2009a). The hypotheses on which it is based are: Circular and deep tunnel, homogeneous and isotropic material around the tunnel, initial lithostatic stresses of a hydrostatic type, linear stress-strain laws in an elastic field and plane strain field.

Unlike the more sophisticated numerical methods that are usually applied in tunneling to simulate both the ground and the support structures (Oreste, 2002; Do *et al*., 2013; 2014a; 2014b), analytical methods, such as the convergence-confinement method, are usually very quick and they allow parametrical analyses to be developed that are useful in the preliminary phase of a study.

Over the years, the convergence-confinement method has also been applied to rock masses, which present a non-linear rupture criterion, as described by (Hoek and Brown, 1980). However, in order to represent these rock masses in the calculation, some simplifying hypotheses are necessary, in particular as far as the strains in the plastic field are concerned (Brown *et al*., 1983; Carranza-Torres and Fairhurst, 2000).

A numerical solution to the convergence-confinement method has been developed in this study. This solution makes it possible to analyze the behavior of tunnels in rock masses, without the need of introducing any added simplifying hypotheses with respect to the basic hypotheses of the method. The utilized approach is the same as that used when a numerical solution is introduced into the convergence-confinement method or into other simple analytical models (Oreste, 2007) so as to be able to consider, for example, bolting (Osgoui and Oreste, 2007; Oreste, 2008; Oreste and Cravero, 2008; Oreste, 2009b; Osgoui and Oreste, 2010; Oreste and Dias, 2012; Oreste, 2013), or when numerical solutions are adopted in simplified analytical models in order to consider more complex aspects, such as the probabilistic variability (Oreste, 2005a) or even to back-analyze the mechanical parameters of the rock, starting from monitoring measurements (Oreste, 2005b).

After having presented the formulation necessary to be able to describe the convergence-confinement curve of a tunnel excavated in rock masses, some significant results on radial displacements and on the thickness of the plastic zone for typical variation intervals of the most influential parameters are reported.

Material and Methods

The radial stresses σ_r and the circumferential stresses σ_ϑ, under elastic behavior conditions of the medium around the tunnel, are described with the following simple Equations 1 and 2 (Ribacchi and Riccioni, 1977; Lembo-Fazio and Ribacchi, 1986; Panet, 1995; Oreste, 2009a):

$$\sigma_\theta = p_0 + \left(p_0 - \sigma_R\right)\cdot\frac{R^2}{r^2} \quad (1)$$

$$\sigma_r = p_0 - \left(p_0 - \sigma_R\right)\cdot\frac{R^2}{r^2} \quad (2)$$

Where:

p_0 = The lithostatic stress that exists at a tunnel depth;
R = The radius of the tunnel;
r = The distance from the center of the tunnel.

In the presence of a plastic zone around the tunnel, whose extension is from R to the plastic radius R_{pl}, the medium beyond the plastic zone ($r>R_{pl}$) has elastic behavior and the radial and circumferential stresses are expressed by the following Equations 3 and 4 (Ribacchi and Riccioni, 1977; Lembo-Fazio and Ribacchi, 1986; Panet, 1995; Oreste, 2009a):

$$\sigma_\theta = p_0 + \left(p_0 - \sigma_{Rpl}\right)\cdot\frac{R_{pl}^2}{r^2} \quad (3)$$

$$\sigma_r = p_0 - \left(p_0 - \sigma_{Rpl}\right)\cdot\frac{R_{pl}^2}{r^2} \quad (4)$$

The by now universally used strength criterion for rock masses is that of Hoek and Brown, in its updated version (Hoek *et al*., 2002):

$$\sigma_{1,\lim} = \sigma_3 + \sigma_{ci}\cdot\left(m_b\cdot\frac{\sigma_3}{\sigma_{ci}} + s\right)^a \quad (5)$$

Where:

$\sigma_{1,\lim}$ = The maximum principle stress upon rupture of the rock mass;
σ_3 = The minimum principle stress (confinement);
σ_{ci} = The uniaxial compression strength of the intact rock;
m_b and s = The strength parameters, which depend on the Geological Strength Index (GSI) (Marinos *et al*., 2005; Hoek *et al*., 2013; Marinos and Hoek, 2000) and on the D parameter: $m_b = m_i\cdot e^{\left(\frac{GSI-100}{28-14\cdot D}\right)}$; $s = e^{\left(\frac{GSI-100}{9-3\cdot D}\right)}$;
D = A parameter that varies between 0 and 1, which considers the disturbance of the rock mass due to the excavation operations (D = 0 for non-disturbed mass; D = 1 for intensely disturbed mass);
m_i = A strength parameter that refers to intact rock and which depends on the typology of the rock;
a = The exponent that is present in Equation 5: $a = 0.5 + \frac{1}{6}\cdot\left(e^{-\frac{GSI}{15}} - e^{-\frac{20}{3}}\right)$.

By deriving the rupture criterion (Equation 5), in relation to the minimum principle stress, Equation 6 is obtained:

$$\frac{d\sigma_{1,\lim}}{d\sigma_3} = 1 + a\cdot m_b\cdot\left(m_b\cdot\frac{\sigma_3}{\sigma_{ci}} + s\right)^{a-1} \quad (6)$$

According to the Mohr-Coulomb criterion, the apparent friction angle of the rock mass depends on the following derivative:

$$sen\varphi = \frac{\frac{d\sigma_{1,\lim}}{d\sigma_3} - 1}{\frac{d\sigma_{1,\lim}}{d\sigma_3} + 1} \quad (7)$$

The Hoek-Brown criterion can be applied to both the peak conditions (elastic behavior limit conditions) and to the residual conditions (conditions in the plastic field) (the subscript p refers to the peak conditions, while the subscript res refers to the residual conditions):

$$\sigma_{1,\lim} = \sigma_3 + \sigma_{ci}\cdot\left(m_{bp}\cdot\frac{\sigma_3}{\sigma_{ci}} + s_p\right)^{a_p} \quad (8)$$

$$\sigma_{1,\lim} = \sigma_3 + \sigma_{ci} \cdot \left(m_{bres} \cdot \frac{\sigma_3}{\sigma_{ci}} + s_{res} \right)^{a_{res}} \quad (9)$$

The radial stress at the plastic radius ($r = R_{pl}$) is obtained by introducing the equivalences of Equation 4 (valid for the elastic behavior zone) with the peak strength criterion of the rock mass (Equation 8):

$$2 \cdot \left(p_0 - \sigma_{Rpl} \right) = \sigma_{ci} \cdot \left(m_{bp} \cdot \frac{\sigma_{Rpl}}{\sigma_{ci}} + s_p \right)^{a_p} \quad (10)$$

From which:

$$p_0 - \sigma_{Rpl} = \frac{\sigma_{ci}}{2} \cdot \left(m_{bp} \cdot \frac{\sigma_{Rpl}}{\sigma_{ci}} + s_p \right)^{a_p} \quad (11)$$

Equation 11 is resolved numerically: If σ_{Rpl} is below zero, no plastic zone will form around the tunnel and the entire rock mass will have elastic behavior. If, instead, σ_{Rpl} is above zero, a plastic zone will form (between $r = R$ and $r = R_{pl}$), inside of which the radial stresses will reduce from $\sigma_r = \sigma_{Rpl}$ for $r = R_{pl}$ to $\sigma_r = p$ for $r = R$, where p stands for the internal pressure of the tunnel.

The trend of the radial stresses in the plastic zone is dictated by the following differential equation (Ribacchi and Riccioni, 1977; Lembo-Fazio and Ribacchi, 1986; Panet, 1995; Oreste, 2009a):

$$\frac{d\sigma_r}{dr} = \frac{\sigma_\theta - \sigma_r}{r} \quad (12)$$

By substituting Equation 9 (strength criterion of the rock mass in residual conditions, which is valid inside the plastic area) in Equation 12 and knowing that the radial stresses are the minimum principle stresses and the circumferential ones are the maximum principle stresses, we obtain:

$$\frac{d\sigma_r}{dr} = \frac{\sigma_{ci} \cdot \left(m_{br} \cdot \frac{\sigma_r}{\sigma_{ci}} + s_r \right)^{a_r}}{r} \quad (13)$$

The circumferential stresses are obtained from the radial ones using Equation 9, which is rewritten in radial and circumferential stress terms as:

$$\sigma_\theta = \sigma_r + \sigma_{ci} \cdot \left(m_{br} \cdot \frac{\sigma_r}{\sigma_{ci}} + s_r \right)^{a_r} \quad (14)$$

As far as the displacements are concerned, the displacement of the tunnel wall u_R, in the absence of a plastic zone, is given by the following expression, which expresses a linear relationship between u_R and p (Ribacchi and Riccioni, 1977; Lembo-Fazio and Ribacchi, 1986; Panet, 1995; Oreste, 2009a):

$$u_R = \frac{1+\nu}{E_p} \cdot \left(p_0 - \sigma_R \right) \cdot R \quad (15)$$

Where:
E_p = The elastic modulus of the rock mass in an elastic field, which can be evaluated on the basis of the GSI (Hoek *et al*., 2002);
ν = The Poisson ratio of the rock mass.

In the presence of a plastic zone, the radial displacement to the plastic radius is obtained through the following equation (Ribacchi and Riccioni, 1977; Lembo-Fazio and Ribacchi, 1986; Panet, 1995; Oreste, 2009a):

$$u_{Rpl} = \frac{1+\nu}{E_p} \cdot \left(p_0 - \sigma_{Rpl} \right) \cdot R_{pl} \quad (16)$$

The evaluation of the radial displacements in the plastic zone is conducted in a correct way if the strains that develop in the elastic-plastic field are known (Ribacchi and Riccioni, 1977; Lembo-Fazio and Ribacchi, 1986; Oreste, 2009a), considering that the maximum principle strain is the circumferential strain ε_ϑ and the minimum principle strain is the radial strain ε_r:

$$\varepsilon_\theta = \varepsilon_{\theta el} + \varepsilon_{\theta pl} = \frac{\left(1-\nu^2\right)}{E_r} \cdot \left[\left(\sigma_\theta - p_0\right) - \frac{\nu}{1-\nu} \cdot \left(\sigma_r - p_0\right) \right] + \lambda \quad (17)$$

$$\varepsilon_r = \varepsilon_{rel} + \varepsilon_{rpl} = \frac{\left(1-\nu^2\right)}{E_r} \cdot \left[\left(\sigma_r - p_0\right) - \frac{\nu}{1-\nu} \cdot \left(\sigma_\vartheta - p_0\right) \right] - \lambda \cdot N_\psi \quad (18)$$

Where: $N_\psi = \frac{1+sen\psi}{1-sen\psi}$
Ψ = The dilatancy expressed in radiants (dilatancy is an angle that can vary between 0 and the friction angle of the material);
$\varepsilon_{\vartheta el}$ and $\varepsilon_{\vartheta pl}$ = The elastic and plastic components of the circumferential strains;
ε_{rel} and ε_{rpl} = The elastic and plastic components of the radial strains;
λ = The plastic multiplier;
E_r = The elastic modulus of the rock mass in the plastic field (which is generally calculated in the same way as for E_p, but

on the basis of a reduced GSI (GSI_{res}), which, as a first approximation, can be estimated as: GSI_{res} = 35+0.5·(GSI-35) for GSI≥35, otherwise GSI_{res} = GSI for GSI<35.

By algebraically summing Equation 18 with Equation 17 multiplied by N_Ψ, Equation 19 is obtained, which connects the total strains in the plastic zone to the existing stresses, in function of the dilatancy, of the Poisson ratio and of the elastic modulus of the rock mass:

$$\varepsilon_\theta \cdot N_\psi + \varepsilon_r = \frac{(1-\nu^2)}{E_{res}} \cdot \left[\begin{array}{l} (\sigma_r - p_0) \cdot \left(1 - N_\psi \cdot \frac{\nu}{1-\nu}\right) \\ +(\sigma_\vartheta - p_0) \cdot \left(N_\psi - \frac{\nu}{1-\nu}\right) \end{array} \right] \tag{19}$$

Since the strains are connected to the radial displacements by the following two congruency relations:

$$\varepsilon_\theta = \frac{u}{r} \tag{20}$$

$$\varepsilon_r = \frac{du}{dr} \tag{21}$$

It is possible to obtain, from Equation 19, the following differential equation which describes the trend of the radial displacements in the plastic zone (Ribacchi and Riccioni, 1977; Lembo-Fazio and Ribacchi, 1986):

$$\frac{du}{dr} = \frac{(1-\nu^2)}{E_r} \cdot \left[\begin{array}{l} (\sigma_r - p_0) \cdot \left(1 - N_\psi \cdot \frac{\nu}{1-\nu}\right) \\ +(\sigma_\vartheta - p_0) \cdot \left(N_\psi - \frac{\nu}{1-\nu}\right) \end{array} \right] - N_\psi \cdot \frac{u}{r} \tag{22}$$

On the basis of the equations reported above, it is possible, utilizing the finite difference method, to integrate Equation 22 in order to obtain the radial displacement on the tunnel wall in the presence of a plastic zone. The procedure foresees the division of the plastic zone around the tunnel into concentric rings and the following calculation steps: σ_r is increased by a very small $\Delta\sigma_r$ value for each chosen internal pressure value σ_R (from 0 to σ_{Rpl}), starting from the tunnel wall. σ_ϑ is then associated to each value of σ_r, using Equation 14 and then the apparent friction angleφ is associated utilizing Equation 6 and 7. From the new calculated radial stress, it is then possible, utilizing Equation 13 rewritten in incremental terms, to determine the increment of radius r and associate this value to the stresses and to the calculated apparent friction angle. It is then necessary to proceed in this way until σ_r reaches the value σ_{Rpl}; at this point, the associated radius corresponds to the searched for plastic radius for that specific value of internal pressure σ_R.

Once the stress situation is known (and the apparent friction angle of the rock mass) for the different distances r inside the plastic zone, it is possible to proceed with the evaluation of the strains, applying the finite difference method to the various previously identified distances r. It is now necessary to start from the just identified plastic radius and to proceed towards the tunnel wall. The dilatancy angle Ψ is calculated in percentage terms with respect to the apparent friction angle that exists at each point. The radial displacement is known for r = R_{pl} from Equation 16 and, through Equation 22, it is possible to also obtain the derivative of the radial displacement in that point. The latter is useful to determine, in incremental terms, the radial displacement for the subsequent value of r and so on until the tunnel wall is reached. In this way, it is possible to obtain the radial displacement of the tunnel wall, for the considered internal pressure σ_R.

Repetition of the procedure for different values of σ_R allows the convergence-confinement curve of the tunnel to be described completely (Fig. 1) as well as the trend of the plastic radius for variations of σ_R (Fig. 2). In particular, if at least 4 points of each of the two diagrams are known, it is possible to precisely obtain both the convergence-confinement curve and the trend of the plastic radius for variations of σ_R.

Results

The proposed solution has been used to find the convergence-confinement curves of various typical conditions of tunnels in rock masses and in order to be able to determine the trend of the plastic radius for variations of σ_R. For this purpose, the following data, which were chosen from intervals of variability of the parameters encountered during the excavation of tunnels in rock masses, were considered: R = 1.5; 3.5; 5.5 m. GSI = 40, 65, 90. m_i = 10, 16, 22. σ_{ci} = 35, 70, 105 MPa. p_0 = 1, 5, 9 MPa.

The other calculation parameters were kept constant costanti (D = 0; ν = 0.3; Ψ = 0 considering the case in which plastic strains evolve at a constant volume (Ribacchi and Riccioni, 1977)). The total number of calculations carried out for all the possible data combinations was 3^5 = 243. The results of the calculations for the points identified in Fig. 1 and 2 are reported in Table 1.

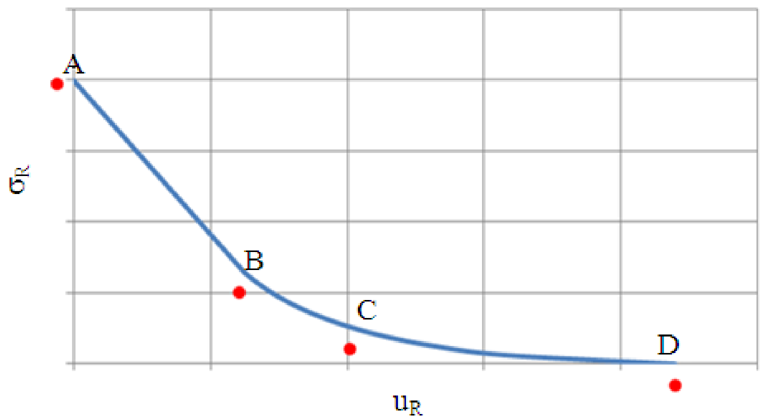

Fig. 1. Convergence-confinement curve of a tunnel in a rock mass: σ_R internal pressure of the tunnel; u_R radial displacement of the tunnel wall. Four points in particular can be identified; (A) $\sigma_R = p_0$ and $u_R = 0$; B) $\sigma_R = \sigma_{Rpl}$ and u_R calculated with Equation 16; (C) $\sigma_R = \sigma_{Rpl}/2$ and u_R calculated by means of the proposed numerical solution; (D) $\sigma_R = 0$ and u_R calculated by means of the proposed numerical solution

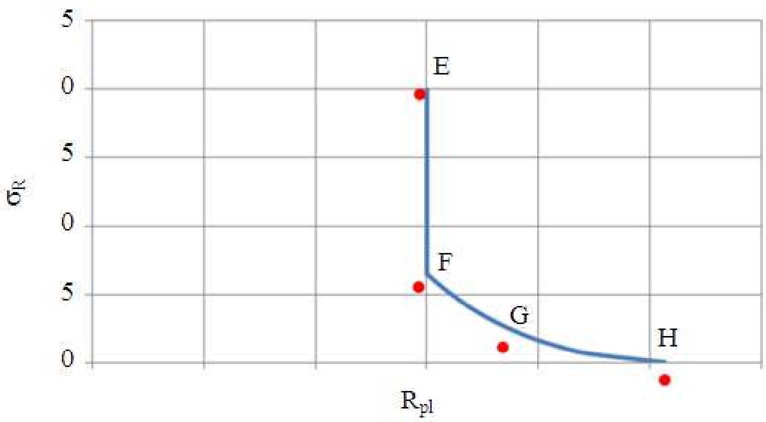

Fig. 2. Trend of the plastic radius R_{pl} for variations of the internal pressure of the tunnel σ_R. Four points in particular can be identified; (E) $\sigma_R = p_0$ and $R_{pl} = R$ (plastic zone not present); (F) $\sigma_R = \sigma_{Rpl}$ and $R_{pl} = R$ (plastic zone not present); (G) $\sigma_R = \sigma_{Rpl}/2$ and R_{pl} calculated by means of the proposed numerical solution; (H) $\sigma_R = 0$ and R_{pl} calculated by means of the proposed numerical solution

Table 1. Resuts of developed calculations

R (m)	P0 (MPa)	GSI	m_i	σ_{ci} (MPa)	σ_{Rpl} (MPa)	u_RC (mm)	u_R D (mm)	R_{pl} G (m)	R_{pl} H (m)
1.5	9	40	16	70	4.86	9.96	95.21	2.37	6.66
1.5	9	40	16	105	4.275	7.46	42.99	2.13	4.65
1.5	9	40	22	35	5.355	16.07	273.75	2.65	10.05
1.5	9	40	22	70	4.383	9.07	59.49	2.17	5.04
1.5	9	40	22	105	3.780	7.00	29.79	1.99	3.73
1.5	9	65	10	35	3.744	4.73	36.81	2.29	5.57
1.5	9	65	10	70	2.619	2.57	8.80	1.92	3.11
1.5	9	65	10	105	1.953	1.93	4.49	1.78	2.42
1.5	9	65	16	35	3.033	3.91	18.96	2.02	3.86
1.5	9	65	16	70	1.998	2.28	5.76	1.78	2.50
1.5	9	65	16	105	1.440	1.77	3.30	1.69	2.09
1.5	9	65	22	35	2.574	3.54	13.09	1.90	3.16
1.5	9	65	22	70	1.620	2.15	4.55	1.71	2.23
1.5	9	65	22	105	1.143	1.69	2.78	1.64	1.93
1.5	9	90	10	35	0.378	0.79	1.17	1.61	1.77
1.5	9	90	10	70	0.000	-	-	-	-

Table 1. Continue

1.5	9	90	10	105	0.000	-	-	-	-
1.5	9	90	16	35	0.261	0.72	0.96	1.57	1.68
1.5	9	90	16	70	0.000	-	-	-	-
1.5	9	90	16	105	0.000	-	-	-	-
1.5	9	90	22	35	0.207	0.69	0.87	1.55	1.63
1.5	9	90	22	70	0.000	-	-	-	-
1.5	9	90	22	105	0.000	-	-	-	-
3.5	1	40	10	35	0.370	2.94	8.30	4.39	6.82
3.5	1	40	10	70	0.252	1.95	3.33	3.98	4.95
3.5	1	40	10	105	0.186	1.61	2.21	3.81	4.34
3.5	1	40	16	35	0.292	2.82	6.08	4.11	5.64
3.5	1	40	16	70	0.187	1.93	2.83	3.82	4.45
3.5	1	40	16	105	0.134	1.60	2.01	3.71	4.06
3.5	1	40	22	35	0.243	2.77	5.14	3.96	5.09
3.5	1	40	22	70	0.150	1.93	2.60	3.75	4.22
3.5	1	40	22	105	0.105	1.60	1.91	3.66	3.92
3.5	1	65	10	35	0.013	0.66	0.69	3.54	3.58
3.5	1	65	10	70	0.000	-	-	-	-
3.5	1	65	10	105	0.000	-	-	-	-
3.5	1	65	16	35	0.009	0.66	0.68	3.53	3.56
3.5	1	65	16	70	0.000	-	-	-	-
3.5	1	65	16	105	0.000	-	-	-	-
3.5	1	65	22	35	0.007	0.66	0.67	3.52	3.54
3.5	1	65	22	70	0.000	-	-	-	-
3.5	1	65	22	105	0.000	-	-	-	-
3.5	1	90	10	35	0.000	-	-	-	-
3.5	1	90	10	70	0.000	-	-	-	-
3.5	1	90	10	105	0.000	-	-	-	-
3.5	1	90	16	35	0.000	-	-	-	-
3.5	1	90	16	70	0.000	-	-	-	-
3.5	1	90	16	105	0.000	-	-	-	-
3.5	1	90	22	35	0.000	-	-	-	-
3.5	1	90	22	70	0.000	-	-	-	-
3.5	1	90	22	105	0.000	-	-	-	-
3.5	5	40	10	35	3.145	23.27	522.87	6.75	29.67
3.5	5	40	10	70	2.620	12.52	95.05	5.37	13.48
3.5	5	40	10	105	2.285	9.46	44.02	4.85	9.55
3.5	5	40	16	35	2.780	18.93	209.58	5.69	17.30
3.5	5	40	16	70	2.220	11.15	52.53	4.77	9.43
3.5	5	40	16	105	1.880	8.75	27.99	4.42	7.26
3.5	5	40	22	35	2.520	17.08	127.02	5.19	12.85
3.5	5	40	22	70	1.945	10.57	37.89	4.48	7.75
3.5	5	40	22	105	1.610	8.46	21.82	4.21	6.25
3.5	5	65	10	35	1.550	4.88	18.60	4.58	7.81
3.5	5	65	10	70	0.925	2.87	5.73	4.04	5.16
3.5	5	65	10	105	0.555	2.21	3.24	3.81	4.32
3.5	5	65	16	35	1.195	4.29	11.81	4.22	6.16
3.5	5	65	16	70	0.670	2.66	4.41	3.85	4.57
3.5	5	65	16	105	0.390	2.11	2.76	3.70	4.04
3.5	5	65	22	35	0.975	4.02	9.15	4.04	5.43
3.5	5	65	22	70	0.525	2.57	3.82	3.77	4.29
3.5	5	65	22	105	0.300	2.06	2.54	3.65	3.90
3.5	5	90	10	35	0.000	-	-	-	-
3.5	5	90	10	70	0.000	-	-	-	-
3.5	5	90	10	105	0.000	-	-	-	-
3.5	5	90	16	35	0.000	-	-	-	-
3.5	5	90	16	70	0.000	-	-	-	-
3.5	5	90	16	105	0.000	-	-	-	-
3.5	5	90	22	35	0.000	-	-	-	-
3.5	5	90	22	70	0.000	-	-	-	-

Table 1. Continue

3.5	5	90	22	105	0.000	-	-	-	-
3.5	9	40	10	35	6.345	60.21	4745.59	8.83	74.67
3.5	9	40	10	70	5.526	28.14	523.43	6.49	25.82
3.5	9	40	10	105	4.977	19.99	195.22	5.66	16.15
3.5	9	40	16	35	5.778	44.02	1296.88	7.02	35.43
3.5	9	40	16	70	4.860	23.24	222.15	5.52	15.55
3.5	9	40	16	105	4.275	17.42	100.31	4.97	10.84
3.5	9	40	22	35	5.355	37.50	638.76	6.18	23.45
3.5	9	40	22	70	4.383	21.17	138.81	5.06	11.75
3.5	9	40	22	105	3.780	16.34	69.50	4.63	8.70
3.5	9	65	10	35	3.744	11.03	85.89	5.33	13.00
3.5	9	65	10	70	2.619	6.00	20.52	4.48	7.25
3.5	9	65	10	105	1.953	4.50	10.48	4.15	5.65
3.5	9	65	16	35	3.033	9.13	44.23	4.72	9.00
3.5	9	65	16	70	1.998	5.33	13.45	4.15	5.83
3.5	9	65	16	105	1.440	4.13	7.70	3.93	4.88
3.5	9	65	22	35	2.574	8.26	30.55	4.43	7.38
3.5	9	65	22	70	1.620	5.02	10.61	3.99	5.20
3.5	9	65	22	105	1.143	3.95	6.49	3.83	4.51
3.5	9	90	10	35	0.378	1.84	2.73	3.75	4.12
3.5	9	90	10	70	0.000	-	-	-	-
3.5	9	90	10	105	0.000	-	-	-	-
3.5	9	90	16	35	0.261	1.68	2.25	3.66	3.91
3.5	9	90	16	70	0.000	-	-	-	-
3.5	9	90	16	105	0.000	-	-	-	-
3.5	9	90	22	35	0.207	1.61	2.04	3.62	3.81
3.5	9	90	22	70	0.000	-	-	-	-
3.5	9	90	22	105	0.000	-	-	-	-
5.5	1	40	10	35	0.370	4.62	13.04	6.90	10.72
5.5	1	40	10	70	0.252	3.07	5.23	6.25	7.77
5.5	1	40	10	105	0.186	2.53	3.48	5.99	6.82
5.5	1	40	16	35	0.292	4.43	9.56	6.45	8.86
5.5	1	40	16	70	0.187	3.04	4.45	6.01	7.00
5.5	1	40	16	105	0.134	2.52	3.16	5.83	6.38
5.5	1	40	22	35	0.243	4.36	8.07	6.23	8.00
5.5	1	40	22	70	0.150	3.03	4.08	5.89	6.63
5.5	1	40	22	105	0.105	2.52	3.00	5.75	6.16
5.5	1	65	10	35	0.013	1.04	1.09	5.56	5.63
5.5	1	65	10	70	0.000	-	-	-	-
5.5	1	65	10	105	0.000	-	-	-	-
5.5	1	65	16	35	0.009	1.04	1.07	5.54	5.59
5.5	1	65	16	70	0.000	-	-	-	-
5.5	1	65	16	105	0.000	-	-	-	-
5.5	1	65	22	35	0.007	1.03	1.05	5.53	5.57
5.5	1	65	22	70	0.000	-	-	-	-
5.5	1	65	22	105	0.000	-	-	-	-
5.5	1	90	10	35	0.000	-	-	-	-
5.5	1	90	10	70	0.000	-	-	-	-
5.5	1	90	10	105	0.000	-	-	-	-
5.5	1	90	16	35	0.000	-	-	-	-
5.5	1	90	16	70	0.000	-	-	-	-
5.5	1	90	16	105	0.000	-	-	-	-
5.5	1	90	22	35	0.000	-	-	-	-
5.5	1	90	22	70	0.000	-	-	-	-
5.5	1	90	22	105	0.000	-	-	-	-
5.5	5	40	10	35	3.145	36.56	821.66	10.61	46.42
5.5	5	40	10	70	2.620	19.68	149.36	8.43	21.18
5.5	5	40	10	105	2.285	14.87	69.17	7.62	15.01
5.5	5	40	16	35	2.780	29.74	329.35	8.95	27.18
5.5	5	40	16	70	2.220	17.52	82.55	7.50	14.82

Table 1. Continue

5.5	5	40	16	105	1.880	13.75	43.99	6.95	11.41
5.5	5	40	22	35	2.520	26.85	199.61	8.16	20.20
5.5	5	40	22	70	1.945	16.61	59.54	7.04	12.19
5.5	5	40	22	105	1.610	13.30	34.29	6.61	9.82
5.5	5	65	10	35	1.550	7.66	29.23	7.20	12.27
5.5	5	65	10	70	0.925	4.51	9.00	6.34	8.10
5.5	5	65	10	105	0.555	3.48	5.09	5.98	6.78
5.5	5	65	16	35	1.195	6.74	18.56	6.63	9.68
5.5	5	65	16	70	0.670	4.19	6.93	6.06	7.17
5.5	5	65	16	105	0.390	3.32	4.34	5.82	6.34
5.5	5	65	22	35	0.975	6.31	14.38	6.36	8.53
5.5	5	65	22	70	0.525	4.04	6.00	5.92	6.73
5.5	5	65	22	105	0.300	3.24	3.99	5.74	6.13
5.5	5	90	10	35	0.000	-	-	-	-
5.5	5	90	10	70	0.000	-	-	-	-
5.5	5	90	10	105	0.000	-	-	-	-
5.5	5	90	16	35	0.000	-	-	-	-
5.5	5	90	16	70	0.000	-	-	-	-
5.5	5	90	16	105	0.000	-	-	-	-
5.5	5	90	22	35	0.000	-	-	-	-
5.5	5	90	22	70	0.000	-	-	-	-
5.5	5	90	22	105	0.000	-	-	-	-
5.5	9	40	10	35	6.345	94.62	7457.35	13.88	117.34
5.5	9	40	10	70	5.526	44.22	822.53	10.19	40.58
5.5	9	40	10	105	4.977	31.42	306.78	8.89	25.38
5.5	9	40	16	35	5.778	69.17	2037.96	11.02	55.68
5.5	9	40	16	70	4.860	36.53	349.09	8.67	24.43
5.5	9	40	16	105	4.275	27.37	157.63	7.81	17.04
5.5	9	40	22	35	5.355	58.92	1003.76	9.72	36.85
5.5	9	40	22	70	4.383	33.27	218.13	7.95	18.47
5.5	9	40	22	105	3.780	25.68	109.22	7.28	13.67
5.5	9	65	10	35	3.744	17.34	134.97	8.38	20.43
5.5	9	65	10	70	2.619	9.43	32.25	7.04	11.39
5.5	9	65	10	105	1.953	7.08	16.46	6.53	8.89
5.5	9	65	16	35	3.033	14.34	69.51	7.42	14.14
5.5	9	65	16	70	1.998	8.38	21.13	6.53	9.16
5.5	9	65	16	105	1.440	6.49	12.10	6.18	7.66
5.5	9	65	22	35	2.574	12.98	48.00	6.96	11.60
5.5	9	65	22	70	1.620	7.88	16.68	6.27	8.16
5.5	9	65	22	105	1.143	6.21	10.20	6.01	7.09
5.5	9	90	10	35	0.378	2.90	4.29	5.89	6.48
5.5	9	90	10	70	0.000	-	-	-	-
5.5	9	90	10	105	0.000	-	-	-	-
5.5	9	90	16	35	0.261	2.65	3.53	5.76	6.14
5.5	9	90	16	70	0.000	-	-	-	-
5.5	9	90	16	105	0.000	-	-	-	-
5.5	9	90	22	35	0.207	2.53	3.20	5.70	5.99
5.5	9	90	22	70	0.000	-	-	-	-
5.5	9	90	22	105	0.000	-	-	-	-

Discussion

Knowledge of the values of $\sigma_R u_R$ and R_{pl} in those points makes it possible to trace diagrams with an elevated degree of accuracy.

In the presence of intermediate input parameters among those analysed, it is also possible to proceed with the interpolation of the results in order to obtain equally the trace of the curves given of Fig. 1 and 2.

Conclusion

The convergence-confinement method is an analytical method that has been frequently used in the scientific community, as it allows the behavior of a tunnel to be analyzed considering the geometrical and mechanical parameters of influence. The simplicity of the method, compared to the more sophisticated numerical methods, leads to reduced calculation times and the development of parametric analyses that are very useful in the preliminary design phase.

In the past, the application of this method to tunnels excavated in rock masses required the introduction of additional simplifying hypotheses to the basic hypotheses of the method, in particular concerning the strains that develop in a plastic field.

A numerical solution to the method is presented in this study. This solution avoids the need for the introduction of added simplifying hypotheses and it proceeds with the evaluation of the strains that develop in the plastic zone around the tunnel in a rigorous way.

The method has been applied to more than 240 cases, which represent the typical variability field of the parameters of influence, in order to be able to obtain the point values of the internal pressure, the radial displacement of the tunnel wall and plastic radius useful to trace the convergence-confinement curve as well as the trend of the plastic radius in function of the internal pressure applied to the tunnel wall.

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